## Limited growth of population model with harvesting:
(or, Limited growth with harvesting)

[ Background: - The effect of harvesting a population on a regular or constant basis is extremely important to many industries such as fishing industry. This model is useful in answering the questions like " will a high harvesting rate destroy the population ? or will a low harvesting rate destroyed the validity of the industry ?

Besides population density or crowding of population, harvesting on a constant basis is to be considered here ]

The Limited population growth model with harvesting is a compartmental model with compartment being the world, town, organization, ocean, Lake, ~~fish~~ pond etc.

In this model, there is an input of population through births and an output through deaths due to crowding of population and harvesting.

By balance law, the word eqn can be written as

$$\left\{\begin{matrix}\text{rate of}\\ \text{change}\\ \text{in}\\ \text{population}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate}\\ \text{of}\\ \text{births}\end{matrix}\right\} - \left\{\begin{matrix}\text{normal}\\ \text{rate}\\ \text{of}\\ \text{deaths}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{death}\\ \text{by}\\ \text{Crowiding}\\ \text{or}\\ \text{density}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate}\\ \text{of}\\ \text{deaths}\\ \text{by}\\ \text{harve-}\\ \text{sting}\end{matrix}\right\}$$

We make the following assumptions

(i) population is sufficiently large so that we can ignore random differences between individuals

(ii) The births and deaths are continuous in time

(iii) Per-capita birth and death rates are constant in time

(iv) We account for increased death rate due to competition of limited resources and the fact that population stabilizes after long period.

(v) We ignore imigration and emigration

(vi) We account for increased deaths due to harvesting.

Formulating the differential Equation:-

Let $u(t)$ be the population in time $t$ and let us consider a population whose initial value is $u_0$ with constant per-capita birth rate $\beta$ and constant per-capita death rate $\alpha$. Let $\gamma u$ be the per capita death rate due to density (where $\gamma$ is constant), as it is ~~pr~~ assumed to be proportional to the population size

We assume that the rate 'h' of harvesting is constant. Thus we get-

rate of births $= \beta u$,

rate of normal deaths $= \alpha u$,

rate of deaths due to density $= \gamma u \cdot u = \gamma u^2$

and, rate of deaths due to harvesting $= h$

Hence, word eqn translate to the diff. eqn

$$\frac{du}{dt} = \beta u - \alpha u - \gamma u^2 - h$$

$$= (\beta - \alpha) u - \gamma u^2 - h$$

$$= r u - \gamma u^2 - h, \quad \text{where } r = \beta - \alpha$$

$$= r u \left(1 - \frac{\gamma u}{r}\right) - h$$

$$\Rightarrow \boxed{\frac{du}{dt} = r u \left(1 - \frac{u}{K}\right) - h} \longrightarrow (1)$$

where $K = \frac{r}{\gamma}$, is the carrying capacity.

Eqn. (1) is the diff. eqn. of limited population growth with harvesting.

Therefore, with the given initial condn $u(0) = u_0$, the IVP for limited population growth with harvesting is given by

$$\boxed{\frac{du}{dt} = r u \left(1 - \frac{u}{K}\right) - h \; ; \; u(0) = u_0} \longrightarrow (2)$$