SOR method :

(Successive Over Relaxation Method) :

(Relaxation Method)

Let us consider the sytem of eqns.

$$\left.\begin{aligned} a_1 x + b_1 y + c_1 z &= d_1 \\ a_2 x + b_2 y + c_2 z &= d_2 \\ a_3 x + b_3 y + c_3 z &= d_3 \end{aligned}\right\} \rightarrow (1)$$

Let us now define reseduals R_x, R_y, R_z by the following relations:

$$\left.\begin{aligned} R_x &= d_1 - a_1 x - b_1 y - c_1 z \\ R_y &= d_2 - a_2 x - b_2 y - c_2 z \\ R_z &= d_3 - a_3 x - a_3 y - a_3 z \end{aligned}\right\} \rightarrow (2)$$

To start with we assume $x = y = z = 0$ and calculate the initial residuals δR_x, δR_y, δR_z. Then the residuals are reduced step by step, by giving some increments to the variables.

We now construct the following table:

Residuals → / ↓ Increments	δR_x	δR_y	δR_z
$\delta x = 1$	$-a_1$	$-a_2$	$-a_3$
$\delta y = 1$	$-b_1$	$-b_2$	$-b_3$
$\delta z = 1$	$-c_1$	$-c_2$	$-c_3$

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From the above equations (2) we see that if x is increased by 1 (keeping y and z constant); R_x, R_y and R_z decrease by a_1, a_2, a_3 respectively. Similarly, effects on the residuals when y and z are given unit increaments are shown in the above table.

At each step, the numerically largest residual is reduced almost to zero. To reduce a particular residual, the value of the corresponding variable is changed. For example to reduce R_x by p, x must be increased by p/a_1 etc.

In case all the residuals have been reduced to almost zero, the increments in x, y, z are added separately to obtain the desired solution.

<u>Note 1.</u> In case the computed values of x, y, z are substituted in (2) and the residuals are calculated and if these residuals are not all negligible, then it means that there is some mistake and the entire process is to be rechecked.

<u>Note 2:</u> Relaxation method (SOR method) can be applied if the diagonal elts. of the co-effts. matrix satisfies the diagonally dominant property.