Gauss-Seidal Iteration Method :

((OR), Method of Successive displacement)

This is the modification of Jaccobi Iteration method.

Consider the system of eqns. be

$$\left.\begin{aligned} a_1x+b_1y+c_1z &= d_1 \\ a_2x+b_2y+c_2z &= d_2 \\ a_3x+b_3y+c_3z &= d_3 \end{aligned}\right\} \rightarrow (1)$$

From (1), we can write.

$$\left.\begin{aligned} x &= \frac{1}{a_1}(d_1-b_1y-c_1z) \\ y &= \frac{1}{b_2}(d_2-a_2x-c_2z) \\ z &= \frac{1}{c_3}(d_3-a_3x-b_3y) \end{aligned}\right\} \rightarrow (2)$$

First Iteration :-

Let us start with initial approximations x_0, y_0, z_0 for x, y, z respectively. Then substituting $y=y_0, z=z_0$ in the first eqn. of (2), we get first approximation of x.

$$x^{(1)} = \frac{1}{a_1}(d_1-b_1y_0-c_1z_0)$$

Then putting $x=x^{(1)}, z=z_0$ in second eqn. of (2), we have the first approximation of y as

$$y^{(1)} = \frac{1}{b_2}(d_2-a_2x^{(1)}-c_2z_0)$$

Next, substituting $x=x^{(1)}, y=y^{(1)}$ in the third eqn. of (2), we get the first approximation of z as

$$z^{(1)} = \frac{1}{c_3}(d_3-a_3x^{(1)}-b_3y^{(1)})$$

2nd Iteration :→

Putting $y = y^{(1)}$, $z = z^{(1)}$ in the 1st eqn. of (2), we get 2nd approx. of x as

$$x^{(2)} = \frac{1}{a_1}\left(d_1 - b_1 y^{(1)} - c_1 z^{(1)}\right).$$

Then putting $x = x^{(2)}$, $z = z^{(1)}$ in second eqn. of (2), we get the second approximation of y as

$$y^{(2)} = \frac{1}{b_2}\left(d_2 - a_2 x^{(2)} - c_2 z^{(1)}\right)$$

Next, putting $x = x^{(2)}$, $y = y^{(2)}$ in the third eqn. of (2), we get the second approximation of z as

$$z^{(2)} = \frac{1}{c_3}\left(d_3 - a_3 x^{(2)} - b_3 y^{(2)}\right)$$

We continue this process untill we get the desired accuracy.

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Note ①: This method can be applied to system of eqns. having more than three unknowns.

Note ②: Like Jaccobi method, Gauss-Seidel method converges for any choice of the initial approximations if in each equation of the system, the absolute value of the largest co-efficient is greater than the sum of the absolute values of all the remaining co-efficients.

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Example: Apply Gauss-Seidal iteration method to solve the system of equations

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Solution: We get from given eqns.,

$$x = \frac{1}{20}(17 - y + 2z) \longrightarrow (1)$$

$$y = \frac{1}{20}(-18 - 3x + z) \longrightarrow (2)$$

$$z = \frac{1}{20}(25 - 2x + 3y) \longrightarrow (3)$$

First Iteration:

Putting $y = y_0 = 0$, $z = z_0 = 0$ in (1), we get

$$x^{(1)} = \frac{1}{20}(17 - y_0 + 2z_0) = \frac{1}{20}(17 - 0 + 0) = 0.8500$$

Putting $x = x^{(1)}$, $z = z_0 = 0$ in (2), we have

$$y^{(1)} = \frac{1}{20}\{-18 - 3(0.8500) + 0\} = -1.0275$$

Putting $x = x^{(1)}$, $y = y^{(1)}$ in (3), we get

$$z^{(1)} = \frac{1}{20}\{25 - 2(0.85) + 3(-1.0275)\} = 1.0109$$

2nd Iteration:

Putting $y = y^{(1)}$, $z = z^{(1)}$ in (1), we get

$$x^{(2)} = \frac{1}{20}\{17 + 1.0275 + 2(1.0109)\} = 1.0025$$

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Putting $x = x^{(2)}$, $z = z^{(1)}$ in **(2)**, we get

$$y^{(2)} = \frac{1}{20}\{-18 - 3(1.0025) + 1.0109\} = -0.9998$$

Again putting $x = x^{(2)}$, $y = y^{(2)}$ in (3), we get

$$z^{(2)} = \frac{1}{20}\{25 - 2(1.0025) + 3(-0.9998)\} = 0.9997$$

Third Iteration :→

Putting $y = y^{(2)}$, $z = z^{(2)}$ in (1), we get

$$x^{(3)} = \frac{1}{20}\left[17 + (0.9998) + 2(0.9997)\right] = 1.0000$$

Putting $x = x^{(3)}$, $z = z^{(2)}$ in (2), we get

$$y^{(3)} = \frac{1}{20}\left[-18 - 3(1) + (0.9997)\right] = -1.0000$$

Again Putting $x = x^{(3)}$, $y = y^{(3)}$ in (3), we get

$$z^{(3)} = \frac{1}{20}\left[25 - 2(1) + 3(-1)\right] = 1.0000$$

Now, the values of 2^{nd} & 3^{rd} iterations are nearly the same, so we stop here.

Hence the solution is $x = 1,\ y = -1,\ z = 1$.

H.W. Ex. Using Gauss - Seidal method, solve the system

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$