Drug Assimilation Model:

Model 2: A course of cold pill

We assume that the drug is delivered to the GI-track continuously, which is reasonable for pills that dissolves slowly in the GI-tract. Thus we assume a constant rate of drug input, I. (e.g. ml of drug per hour). Also, since the pill dissolves slowly, we assume that initially there is no drug in the GI-tract.

Let $u(t)$ and $v(t)$ are the amount of drug in the G.I-tract and in bloodstream at time t respectively.

By balance law, the word eqns. are given by

$$\left\{\begin{matrix}\text{rate of change}\\ \text{of drug}\\ \text{in GI-tract}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of}\\ \text{drug}\\ \text{intake}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{drug leaves}\\ \text{GI-tract}\end{matrix}\right\}$$

and

$$\left\{\begin{matrix}\text{rate of change}\\ \text{of drug}\\ \text{in blood}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of}\\ \text{drug}\\ \text{enters blood}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{drug}\\ \text{leaves blood}\end{matrix}\right\}$$

Transforming the above two word eqns. into differential eqns. (i.e. mathematical model), we get

$$\frac{du}{dt} = I - m_1 u \; ; \quad u(0) = 0 \longrightarrow (1)$$

and
$$\frac{dv}{dt} = m_1 u - m_2 v \; ; \quad v(0) = 0 \longrightarrow (2)$$

where I is a positive constant representing the rate of ingestion of the drug (in grams of unit of time).

(Here we note that here $u(0) = 0$, whereas in the previous model $u(0) = u_0$)

Now: To find soln. of (1) & (2):

From (1), we get

$$\frac{du}{dt} + m_1 u = I$$

which is linear.

Its I.F. $= e^{\int m_1 dt} = e^{m_1 t}$

$\therefore$ soln. is

$$u \cdot e^{m_1 t} = \int e^{m_1 t} \cdot I \, dt + c \text{ (constant)}$$

$$u \cdot e^{m_1 t} = I \frac{e^{m_1 t}}{m_1} + c \longrightarrow (3)$$

which is G.S. of (1).

Now, using The initial condⁿ. $u(0)=0$, we get

$$c = -\frac{1}{m_1}$$

Putting this value of ~~c~~ in (3),

$$u e^{m_1 t} = \frac{I}{m_1} e^{m_1 t} - \frac{I}{m_1}$$

$$= \frac{I}{m_1}\left(e^{m_1 t} - 1\right)$$

$$\Rightarrow u = \frac{I}{m_1}\left(1 - e^{-m_1 t}\right) \rightarrow (4)$$

which is a particular solⁿ. of (1) for given initial condⁿ.

To solve (2), Putting The value of u from (4) into (2), we get

$$\frac{dv}{dt} = m_1 \cdot \frac{I}{m_1}\left(1 - e^{-m_1 t}\right) - m_2 v$$

$$= I\left(1 - e^{-m_1 t}\right) - m_2 v$$

$$\Rightarrow \frac{dv}{dt} + m_2 v = I\left(1 - e^{-m_1 t}\right)$$

which is linear.

Its $I.F. = e^{\int m_2 dt} = e^{m_2 t}$

$\therefore$ sol^n. is

$$v.e^{m_2 t} = \int e^{m_2 t}.I(1-e^{-m_1 t})dt + c_1 \text{ (const.)}$$

$$\Rightarrow v.e^{m_2 t} = I\left[\frac{e^{m_2 t}}{m_2} - \frac{e^{(m_2-m_1)t}}{m_2-m_1}\right] + c_1 \longrightarrow (5)$$

Which is the G.S. of (2).

For its Particular sol^n, using initial condition $v(0) = 0$ in (5), we get

$$0.e^0 = I\left[\frac{e^0}{m_2} - \frac{e^0}{m_2-m_1}\right] + c_1$$

$$\Rightarrow 0 = I\left(\frac{1}{m_2} - \frac{1}{m_2-m_1}\right) + c_1$$

$$\Rightarrow c_1 = -I\left[\frac{1}{m_2} - \frac{1}{m_2-m_1}\right]$$

Putting this value of c_1 in (5), we get

$$v(t) = I\left[\frac{1}{m_2} - \frac{e^{-m_1 t}}{m_2-m_1}\right] - I\left[\frac{1}{m_2} - \frac{1}{m_2-m_1}\right]e^{-m_2 t}$$

$$= \frac{I}{m_2}\left[1-e^{-m_2 t}\right] - \frac{I}{m_2-m_1}\left[e^{-m_1 t} - e^{-m_2 t}\right]$$

$$= \frac{I}{m_2} - \frac{I}{m_2}e^{-m_2 t} - \frac{I}{m_2-m_1}\left[e^{-m_1 t} - e^{-m_2 t}\right]$$

or,

$$V(t) = \frac{I}{m_2} - \frac{I}{m_2(m_2-m_1)}\left\{\cancel{m_2 e^{-m_2 t}} - m_1 e^{-m_2 t} + m_2 e^{-m_1 t} - \cancel{m_2 e^{-m_2 t}}\right\}$$

$$= \frac{I}{m_2} - \frac{I}{m_2(m_2-m_1)}\left\{m_2 e^{-m_1 t} - m_1 e^{-m_2 t}\right\}$$

$$\Rightarrow V(t) = \frac{I}{m_2}\left[1 - \frac{1}{m_2-m_1}\left(m_2 e^{-m_1 t} - m_1 e^{-m_2 t}\right)\right] \longrightarrow (6)$$

which is the required particular solution of (2) with initial given cond^{ns}.

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