Drug Assimilation Model :

We investigate two simple models of cold pill assimilation into the bloodstrem, viz.

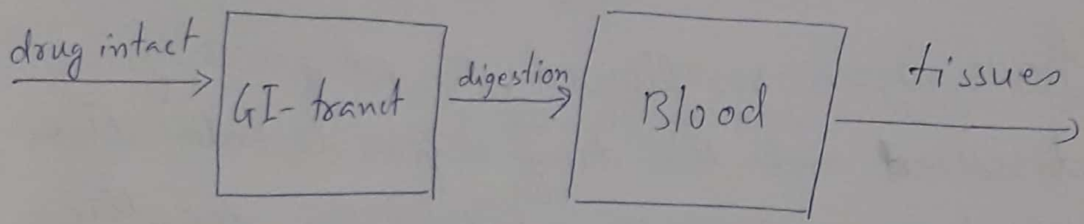
① a single cold pill model

(ii) a course of cold pills model.

[Background:- Whenever we use pills or drug, it dissolves in the gastrointestinal tract (GI-tract) and each ingredient is diffused into the bloodstream. They are carried to the locations in which they act and are removed from the blood by the kidneys and the liver. The assimilation and removal may occur at different rates for the different ingredients of the same pill. In this section, with the help of related mathematical model, which will be formulated, and also some examples and case study, we see that different drugs are absorbed into, and extracted from, the blood at very different rates. Further, in the case ~~of~~ study on alcohol absorption, we see how different body masses and the sex of an individual can radically modify the effects of alcohol.]

Drug assimilation model:

We can consider drug assimilation model as a compartmental model with two compartments, corresponding to the GI-tract (Gastro-Intestinal tract) and the ~~other~~ blood stream. The G.I.tract compartment has a single input and output and the bloodstream compartment has a single input and output.



Using balance law, the word eqns are given by

$$\left\{\begin{array}{c}\text{rate of change}\\ \text{of drug}\\ \text{in GI-tranct}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of}\\ \text{drug}\\ \text{intake}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of drug}\\ \text{leaves G.I.tract}\end{array}\right\}$$

and

$$\left\{\begin{array}{c}\text{rate of change}\\ \text{of drug}\\ \text{in blood}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of}\\ \text{drug}\\ \text{enters blood}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of}\\ \text{drug}\\ \text{leaves blood}\end{array}\right\}$$

We consider two models:—

(i) A single cold pill where there is no ingestion of the drug except that which occurs initially,

(ii) A course of pills where the drug intake is assumed to occur continuously.

Model (1): A single cold pill:

In GI-tract, we consider the pill to have been swallowed, the pill quickly dissolved and the drug begins to enter the bloodstream from the G.I-tract. So, for the G.I.-tract there is only an output term.

Let $u(t)$ be the amount of drug in the GI-tract and $v(t)$ the amount in the bloodstream at time t.

We assume the output rate is proportional to the GI-tract drug concentration, which is therefore proportional to the amount of drug in the bloodstream. So by balance law, we get

$$\boxed{\frac{du}{dt} = -m_1 u; \quad u(0) = u_0} \longrightarrow (1)$$

where u_0 is the amount of a drug in the pill, our initial condition, and m_1 is the const. of proportionality (the rate constant). We assume that the instant the pill enters the GI-tract at $t=0$, it dissolves instantaneously so $v(0) = 0$

In the bloodstream, the initial amount of the drug is zero, so $v(0) = 0$. The level in the bloodstream increases as the drug diffuses from the GI-tract and decreases as the kidneys and liver remove it. Hence, using balance law, we get

$$\boxed{\frac{dv}{dt} = m_1 u - m_2 v \,, \quad v(0) = 0} \longrightarrow (2)$$

where m_2 another positive constant of proportionality and $m_1 \neq m_2$.

The differential Eqns. (1) & (2) are the required model of drug assimilation for a single cold pill. //

<u>Solution of the model</u> :$\longrightarrow$

From eqn. (1), we get

$$\frac{du}{dt} = -m_1 u$$

$$\Rightarrow \frac{du}{u} = -m_1 \, dt$$

$$\Rightarrow \int \frac{du}{u} = -\int m_1 \, dt$$

$$\Rightarrow \log u = -m_1 t + \log A \,, \quad A \text{ is const.}$$

$$\Rightarrow \log \frac{u}{A} = -m_1 t$$

$$\Rightarrow \boxed{u = A e^{-m_1 t}} \longrightarrow (3)$$

which is the G.S. of (1).

Using initial condition $u(0) = u_0$ in (3), we get

$$u_0 = A e^0$$

$$\Rightarrow A = u_0.$$

Putting this in (3),

$$\boxed{u(t) = u_0 e^{-m_1 t}} \longrightarrow (4)$$

which is a particular solⁿ of (1).

Now, putting this value of $u(t)$ from (4) in the equation (2), we get

$$\frac{dv}{dt} = m_1 u_0 e^{-m_1 t} - m_2 v$$

$$\Rightarrow \frac{dv}{dt} + m_2 v = m_1 u_0 e^{-m_1 t} \longrightarrow (5)$$

Which is a linear differential Equation of 1st order 1st degree.

Its

$$I.F. = e^{\int m_2 dt} = e^{m_2 t}$$

∴ soln. of (5) is

$$v . e^{m_2 t} = \int e^{m_2 t} . m_1 u_0 e^{-m_1 t} dt + c$$

$$= m_1 u_0 \int e^{(m_2 - m_1)t} dt + c$$

$$\Rightarrow \boxed{v . e^{m_2 t} = m_1 u_0 \frac{e^{(m_2 - m_1)t}}{m_2 - m_1} + c} \rightarrow (6)$$

which is the G.S. of (2).

Using the initial condition $v(0) = 0$, we get

$$0 \cdot e^0 = m_1 u_0 \frac{e^0}{m_2 - m_1} + c$$

$$\Rightarrow c = -\frac{m_1 u_0}{m_2 - m_1}$$

$$\therefore \quad v e^{m_2 t} = m_1 u_0 \frac{e^{(m_2 - m_1)t}}{m_2 - m_1} - \frac{m_1 u_0}{m_2 - m_1}$$

$$= \frac{m_1 u_0}{m_2 - m_1} \left[e^{(m_2 - m_1)t} - 1 \right]$$

$$\Rightarrow v = \frac{m_1 u_0}{m_2 - m_1} \left[e^{-m_1 t} - e^{-m_2 t} \right] \rightarrow (7)$$

which is the required particular solution of (2) with given initial condn.