Case study: Lake Burley Griffin:

The information for the case study is adapted from the research paper of Burgess and Olive (1975).

Lake Burley Griffin in Canberra, the capital city of Australia, was created artificially in 1962 for both recreational and aesthetic purposes. In 1974, the public health authorities indicated that pollution standard set down for safe recreational use were being violated and that this was attributed to the sewage works in Queanbeyan upstream (or rather the dischange of untreated sewage into the lake's feeder river).

After extensive measurements of pollution levels taken in the 1970s it was established that, while the sewage plants (of which there are three above the lake) certainely exacerbated the problem, there were significant contributions from rural and urban runoff as well, particularly during summer rainstroms. These contributed to dramatic increases in pollution levels at times were totally responsible for lifting the concentration levels above the safety limits. As a point of interest, Queanbeyan (where the sewage plants are situated) is in the state of New South Wales (NSW) while the lake is in the Australian Capital Territory, and, although they are at a ten-minute drive apart, the safety levels/standards for those who swim in NSW are different from the standards for those who swim in the Capital Teritorry.

In 1974, the mean concentration of the bacteria faecal coliform count was approximately 10^7 bacteria per m^3 at the point where the river feeds into the lake. The safety threshold for this faecal coliform count in the water is such that for contact ~~reciprocal~~ recreational sports no more than 10% of total samples over a days period should exceed 4×10^6 bacteria per m^3.

Given that the lake was polluted, it is of interest to examine how, if sewage management were improved, the lake would flush out and if and when the pollution level would drop below safety threshold.

The system can be modelled, very simply, under a few assumptions.

Let us make the following assumptions.

i) Flow ~~f~~ into the lake is assumed ~~to~~ equal to flow out of the lake.

(ii) Volume V of the lake is constant and is approximately $28 \times 10^6\ m^3$.

(iii) The lake is well mixed in the sense that the pollution concentration throughout will be taken as constant

~~Under the above~~ assumptions

Under the above assumptions, the differential eqn for the pollutant concentration is (as before)

$$\frac{dc}{dt} = \frac{f}{V}(c_{in} - c) \; ; \; c(0) = c_0 \longrightarrow (1)$$

& the G. solution of (1) is given by

$$c(t) = c_{in} - (c_{in} - c_0)\, e^{-\frac{f}{V}t} \longrightarrow (2)$$

Since only fresh water is entering into the lake, $c_{in} = 0$.

$$\therefore \; c(t) = c_0 \, e^{-\frac{f}{V}t} \longrightarrow (3).$$

Given that

$$c(t) = 4 \times 10^6, \; V = 28 \times 10^6 \, m^3,$$

$$f = 4 \times 10^6, \; C_0 = 10^7$$

$$\therefore (3) \Rightarrow 4 \times 10^6 = 10^7 . \, e^{-\frac{4 \times 10^6}{28 \times 10^6} t}$$

$$\Rightarrow 0.4 = e^{-\frac{1}{7}t}$$

$$\Rightarrow \log_e(0.4) = -\frac{1}{7}t$$

$$\Rightarrow t = -7 \times \log_e(0.4)$$

$$= -7 \times (-0.91629)$$

$$= 6.71403$$

$$\simeq 6.7$$

Hence, the lake will take approximately 6.7 months for the pollution level to drop below the safety threshold. //