

Q. Find the root of

$$x^2 - 5x + 2 = 0$$

correct to the five decimal places by Newton-Raphson method.

Soln. Let The given equation is

$$f(x) = x^2 - 5x + 2 = 0 \quad \text{--- (1)}$$

The given eqn (1) has two roots. It can be seen by trial or from the graph, that the approximate value of the roots are 0, 4.

Let $f(x) = x^2 - 5x + 2$

$$\therefore f'(x) = 2x - 5$$

By Newton-Raphson formula, we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \\ = x_n - \frac{x_n^2 - 5x_n + 2}{2x_n - 5}$$

$$\text{i.e., } x_{n+1} = \frac{x_n^2 - 2}{2x_n - 5} \quad \text{--- (2)}$$

Putting $n=0$, we get-

$$x_1 = \frac{x_0^2 - 2}{2x_0 - 5} \quad \text{[starting with } x_0 = 0\text{]} \\ = \frac{-2}{-5} \\ = 0.4$$

Again, we have, putting $n=1$, in (2),

$$\begin{aligned}
 x_2 &= \frac{x_1^2 - 2}{2x_1 - 5} \\
 &= \frac{(0.4)^2 - 2}{2 \times (0.4) - 5} \\
 &= \frac{-1.84}{-4.20} \\
 &= 0.438095
 \end{aligned}$$

Putting, $x = 2$ in (2), we get-

$$\begin{aligned}
 x_3 &= \frac{x_2^2 - 2}{2x_2 - 5} \\
 &= \frac{(0.438095)^2 - 2}{2 \times (0.438095) - 5} \\
 &= \frac{-0.1808072771}{-4.12381} \\
 &= 0.0438447
 \end{aligned}$$

Also, putting $n=3$ in (2), we get-

$$\begin{aligned}
 x_4 &= \frac{x_3^2 - 2}{2x_3 - 5} \\
 &= \frac{(0.0438447)^2 - 2}{2 \times (0.0438447) - 5} \\
 &= \frac{-1.8077642282}{-4.123106} \\
 &= 0.4384471872 \\
 &= 0.438447
 \end{aligned}$$

Here we see that $x_3 = x_4 = 0.438447$; so that one of the real roots is 0.438447.

(28)

For the second real root, we start with $x_0 = 1$. Then

$$x_1 = \frac{x_0^2 - 2}{2x_0 - 5} = \frac{1^2 - 2}{2 \times 1 - 5} = \frac{-1}{-3} = \frac{1}{3} = 0.333333$$

Further,

$$x_2 = \frac{x_1^2 - 2}{2x_1 - 5} = \frac{(0.333333)^2 - 2}{2 \times 0.333333 - 5}$$

$$= \frac{0.111111 - 2}{0.666666 - 5} = \frac{-1.888889}{-4.333334} = 4.357143$$

$$x_3 = \frac{x_2^2 - 2}{2x_2 - 5} = \frac{(4.357143)^2 - 2}{2 \times 4.357143 - 5}$$

$$= \frac{18.987654 - 2}{8.714286 - 5} = \frac{16.987654}{3.714286} = 4.571429$$

$$x_4 = \frac{x_3^2 - 2}{2x_3 - 5} = \frac{(4.571429)^2 - 2}{2 \times (4.571429) - 5}$$

$$= \frac{20.897959 - 2}{9.142858 - 5} = \frac{18.897959}{4.142858} = 4.561553$$

$$= 4.5615528128$$

$$\approx 4.561553$$

Obviously, $x_3 = x_4$ so that the
another real root is 4.561553.

x