Half Life :-

Half life of a radioactive isotope is the time required for half of it to decay. It is denoted by τ (tao).

Relation between τ and decay constant k :

We have exponential decay model

$$\frac{dN(t)}{dt} = -k\,N(t) \longrightarrow (1)$$

Solving we get

$$N = N_0 e^{-kt} \longrightarrow (2)$$

where $N(0) = N_0$.

Now, we put $t = \tau$ and $\frac{N}{N_0} = \frac{1}{2}$ in (2) & get

$$\frac{1}{2} = e^{-k\tau} \Rightarrow e^{-k\tau} = 2$$

$$\Rightarrow +k\tau = \log_e 2$$

$$\Rightarrow k = \frac{\log_e 2}{\tau} \quad \left(\text{or, } \tau = \frac{\log_e 2}{k}\right).$$

Note: ① Rel^n. bet^n. Natural & common logarithm:

$$\log_e(x) = \log_{10}(x) \times \log_e 10$$

$$= \log_{10}(x) \div \frac{1}{\log_{10} e}$$

$$= \log_{10}(x) \div \frac{1}{\log_{10}(2.71828)} \quad [\because e \cong 2.71828]$$

$\ln(x)$
$= \log_e(x) \rightarrow$ Natural logarithm
$\log_{10}(x) \rightarrow$ Common "

Note: ②: $\tau = \frac{1}{k}\left(\log_e 2\right) = \frac{1}{k}\,\frac{\log_{10} 2}{\log_{10}(2.71828)}$

$$= \frac{1}{k}\,\frac{0.30103}{0.4343}, \text{ by log table}$$

Note ③: For C-14,
$k = 0.0001216$; Hence $\tau = \frac{1}{0.0001216} \times \frac{0.30103}{0.4343} \cong 5719$ years.

Example: Carbon extracted from an ancient skull contained only one-sixth as much C-14 as carbon extracted from present day bone. Determine the age of the skull. (given decay const(k) of C-14 is = 0.0001216).

Soln We have the decay model

$$\frac{dN}{dt} = -kN \longrightarrow (1)$$

& its soln. is

$$N = N_0 e^{-kt}, \quad \text{where } N(0) = N_0. \longrightarrow (2)$$

Now, ATQ, we put $\frac{N}{N_0} = \frac{1}{6}$ & $k = 0.0001216$ in (2)

& get,

$$\frac{1}{6} = e^{-0.0001216t}$$

$$\Rightarrow -0.0001216t = \log_e\left(\frac{1}{6}\right) = \log_{10}\left(\frac{1}{6}\right) \times \log_e(10)$$

$$= \log_{10}\left(\frac{1}{6}\right) \times \frac{1}{\log_{10} e}$$

$$\Rightarrow (-0.0001216t)\log_{10} e = \log_{10}(1) - \log_{10}(6)$$

$$\Rightarrow (-0.0001216t) \times (0.4343) = 0 - 0.778$$

(using log table)

$\log_{10} e$
$= \log_{10}(2.71828)$
$= 0.4343$
by log table

$$\Rightarrow t = \frac{0.778}{0.0001216 \times 0.4343}$$

$$= 14731.812838$$

$$\simeq 14731.8$$

Thus, the age of skull = 14731.8 years

x

Example: → Radium decomposes at a rate proportional to the amount present. If half of the original amount disappears in 1600 years, find the percentage lost in 100 years.

Soln. Let, $m(t)$ = the mass at time t years.

Then we get the decay model as

$$\frac{d\,m(t)}{dt} = -k\,m(t) \longrightarrow (1)$$

$$\Rightarrow \frac{dm}{m} = -k\,dt$$

Integrating,

$$\int \frac{dm}{m} = -k\int dt + \log_e c$$

$$\Rightarrow \log_e m = -kt + \log_e c$$

$$\Rightarrow \log_e \frac{m}{c} = -kt$$

$$\Rightarrow \frac{m}{c} = e^{-kt}$$

$$\Rightarrow m = c\,e^{-kt} \longrightarrow (2)$$

Now, if initially, when $t=0$, $m = m_0$ i.e., $m(0)=m_0$, then from (2),

$$m_0 = c\,e^{-k.0} \Rightarrow m_0 = c$$

∴ Putting in (2),

$$m(t) = m_0\,e^{-kt} \longrightarrow (3)$$

Now, given $m = \frac{1}{2}m_0$, when $t = 1600$.

$$\therefore (3) \Rightarrow \frac{1}{2}m_0 = m_0\,e^{-k(1600)}$$

$$\Rightarrow 2 = e^{1600k} \longrightarrow (4)$$

When $t = 100$, then from (3) ~~i.e. from $m = m_0 \bar{e}$~~

$$m = m_0 \bar{e}^{100k}$$

$$= m_0 \cdot \left(e^{1600k}\right)^{\frac{-1}{16}}$$

$$= m_0 (2)^{-\frac{1}{16}} \quad \text{, from (4)}$$

$$= m_0 \frac{1}{2^{1/16}}$$

$$= m_0 \frac{1}{1.0442737824} \quad \text{, using scientific calculator}$$

$$= m_0 (0.9576032807) \quad \text{"}$$

$$\simeq m_0 (0.96)$$

$$\Rightarrow m = 96\% \text{ of } m.$$

Hence loss $= (100 - 96)\% = 4\%$