Q. A real root of the equation

$$x^3 - 5x + 1 = 0$$

lies in the interval (0, 1). Perform four iterations of the secant method.

Soln. Let

$$f(x) = x^3 - 5x + 1$$

Let the given interval (0, 1) be (x_1, x_2). i.e. $x_1 = 0$, $x_2 = 1$

Now,

$$\therefore f(x_1) = f(0) = 1$$

$$f(x_2) = f(1) = 1 - 5 + 1 = -3$$

We have the general formula of secant method

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}, \quad n \geq 2 \rightarrow (1)$$

Putting $n = 2$, we have the 1st approx. of the root

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{0 \cdot (-3) - 1 \cdot 1}{-3 - 1}$$

$$= \frac{1}{4}$$

$$= 0.25$$

Now, $f(x_3) = (0.25)^3 - 5 \times (0.25) + 1$

$$= 0.015625 - 1.25 + 1$$

$$= -0.234375.$$

Again, putting $n=3$ in the formula (1), we get 2nd approx.

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{1.(-0.234375) + 0.25 \times 3}{-0.234375 + 3}$$

$$= \frac{0.515625}{2.765625} = 0.18644$$

Also, $f(x_4) = (0.18644)^3 - 5 \times (0.18644) + 1 = 0.07428$

∴ Putting $n=4$ in (1), we get 3rd appror. of the root

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{0.25 \times (0.07428) - 0.18644 \times (-0.234375)}{0.07428 + 0.234375}$$

$$= \frac{0.062266875}{0.308655} = 0.20174$$

Also, $f(x_5) = (0.20175)^3 - 5 \times (0.20174) + 1 = 0.000488$

∴ Putting $n=5$ in (1), we get the 4th approximation of the root (i.e. required fourth iteration)

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{0.18644 \times 0.000488 - 0.20174 \times 0.07428}{0.000488 - 0.07428}$$

$$= \frac{-0.014894264 5}{-0.073792} = 0.20184$$

which is the required root. #

Q. Estimate the root of the eqn.

$$\cos x - xe^x = 0$$

using the <u>secant method</u> with initial estimate at $x_1 = 0.5$ and $x_2 = 1$.

<u>Soln.</u> Let-

$$f(x) = \cos x - xe^x \longrightarrow (1)$$

$$\therefore f(x_1) = f(0.5) = \cos(0.5) - (0.5)e^{0.5}$$

$$= \cos\left(\frac{180^\circ}{\pi} \times 0.5\right) - (0.5)e^{0.5}$$

$$= 0.87758 - 0.82436 = 0.0532$$

$$f(x_2) = f(1) = \cos(1) - x e^1$$

$$= \cos\left(\frac{180^\circ \times 1}{\pi}\right) - 1e^1$$

$$= 0.5403023 - 2.7182818 = -2.17798$$

Now, we have the general formula of secant method as

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})} \quad , n \geq 2 \rightarrow (2)$$

Putting $n = 2$, we get-

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{(0.5)(-2.17798) - 1(0.0532)}{-2.17798 - 0.0532}$$

$$= \frac{-1.14219}{-2.23118} = 0.511921426$$

$$= 0.51192$$

Now, $f(x_3) = \cos(0.5119) - 0.5119 \times e^{0.5119}$

$= 0.87182 - 0.5119 \times 1.668458$

$= 0.87182 - 0.85408$

$= 0.01774$

Again, putting $n = 3$, in (1), we get-

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{1 \times (0.01774) + 0.51192 \times (2.17798)}{0.01774 + 2.17798}$$

$$= \frac{1.13235152 16}{2.19572} = 0.5157$$

Again, $f(x_4) = \cos(0.5157) - 0.5157 \times e^{0.5157}$

$= 0.8699477342 - 0.8637$ $= 0.0062479$

$= 0.006248$

$= 0.00525$

$n = 4 \Rightarrow$

$$\therefore x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{0.5119 \times 0.00625 - 0.5157 \times 0.01774}{0.00625 - 0.01774}$$

$$= \frac{-0.005949}{-0.01149} = 0.5178$$

Again, $f(x_5) = \cos(0.5178) - 0.5178 \times e^{0.5178}$

$= 0.868910215 - 0.5178 \times 1.67833 = -0.00012$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)} = \frac{0.5157 \times (-0.00012) - 0.5178 \times 0.00525}{-0.00012 - 0.00525}$$

$$= \frac{-0.00278055}{-0.00537} = 0.5178 \leftarrow \text{Ans}$$