Test OF SIGNIFICANCE OF LSE

After estimating the parameters of the linear regression relationship of any economic postulates, we are concern to establishe criteria for judging the goodness of parameter estimated. The test of significance of the estimates, therefore, implies the criteria for judging the goodness of the estimates.

We have three different criteria —

1. Theoritical criterian
2. Stastical Criterion.
3. Econometric Criterion.

But, we are mainly concern with the stastical criteria. It implies two tests for judging the goodness of the estimates.

1. The square of the correlation coefficient test.
2. Standard Error Test.

(I) TEST OF GOODNESS OF THE ESTIMATE WITH THE HELP OF r^2

or,

THE SQUARE OF THE CORRELATION COEFFIENT TEST.

After estimation of the parameters and determination of the regression line, we should have to know how good is the fit of this line to the sample observations of Y and X. This requires to know or to measure the dispersion of the observations around the regression line. The regression line will be best fitted if the observations are very close to the reg. line. Therefore, the goodness of fit depends on the closeness of observations to the regression line.

The measure of the goodness of fit is given by the square of correlation coefficient. It gives the percentage of the total variation of the dependent variables that can be explained by the explanatory variable.

For this purpose, we are to estimate or formulate the following formulae

$$\bar{X} = \frac{\Sigma X_t}{n},$$

$$\bar{Y} = \frac{\Sigma Y_t}{n} \quad \text{and}$$

the estimated regression line

$$\hat{Y}_t = \hat{\alpha} + \hat{\beta} X_t$$

We know that the total variation of the dependent variables consists of two parts - one is the explained variation and other is the unexplained variation which is due to the existence of random term in any economic relationship.

Total variation in $Y = \sum_{t=1}^{n} (Y_t - \bar{Y})^2 = \Sigma y_t^2 \longrightarrow (1)$

The deviation of the regressed value is given by the difference between the regression line and the mean value of the actual observations. Thus, the explained variation in Y

$$= \Sigma(\hat{Y}_t - \bar{Y})^2 = \Sigma \hat{y}_t^2 \longrightarrow (2)$$

The unexplained variation is given by the squares of the residuals. Therefore,

$$\text{Unexplained Variation in } Y = \sum_{t=1}^{n}(Y_t - \hat{Y}_t)^2 = \Sigma u_t^2 \longrightarrow (3)$$

Thus, we have three types of deviations —

(i) $y_t = Y_t - \bar{Y}$ = deviation of the Y_t i.e., observation from its mean.

(ii) $\hat{y}_t = \hat{Y}_t - \bar{Y}$ = deviation of $\hat{Y}_t$ i.e., estimated regression line from the actual mean.

(iii) $\hat{u}_t = Y_t - \hat{Y}_t$ = deviation of the actual observations from the estimated regression line.

Now, combining the above expressions, we can have—

$$y_t = Y_t - \bar{Y}$$

$$\Rightarrow Y_t = y_t + \bar{Y} \longrightarrow (4)$$

$$\hat{y}_t = \hat{Y}_t - \bar{Y}$$

$$\Rightarrow \hat{Y}_t = \hat{y}_t + \bar{Y} \longrightarrow (5)$$

Now, putting (4) and (5) in $u_t = Y_t - \hat{Y}_t$, we get,

$$u_t = Y_t - \hat{Y}_t$$

$$= y_t + \bar{Y} - \hat{y}_t - \bar{Y}$$

$$= y_t - \hat{y}_t \longrightarrow (6)$$

$$\Rightarrow y_t = \hat{y}_t + u_t \longrightarrow (6.a)$$

Now, taking the sum of the squares of both side of ⑥, we get,

$$\Sigma y_t^2 = \Sigma(\hat{y}_t + u_t)^2$$

$$= \Sigma(\hat{y}_t^2 + 2\hat{y}_t u_t + u_t^2)$$

$$= \Sigma\hat{y}_t^2 + 2\Sigma\hat{y}_t u_t + \Sigma u_t^2$$

$$\Rightarrow \Sigma y_t^2 = \Sigma\hat{y}_t^2 + \Sigma u_t^2 \qquad (\because \Sigma\hat{y}_t u_t = 0)$$

$\Rightarrow$ Total variation = Explained Variation + Unexplained Variation $\longrightarrow$ ⑦

N.B.

Prove that $\Sigma\hat{y}_t u_t = 0$

We have, $\hat{Y}_t = \hat{\alpha} + \hat{\beta}X_t$ and

$$\bar{Y} = \hat{\alpha} + \hat{\beta}\bar{X}$$

Again, $\hat{y}_t = \hat{Y}_t - \bar{Y}$

$$= \hat{\alpha} + \hat{\beta}X_t - (\hat{\alpha} + \hat{\beta}\bar{X})$$

$$= \hat{\alpha} + \hat{\beta}X_t - \hat{\alpha} - \hat{\beta}\bar{X}$$

$$= \hat{\beta}X_t - \hat{\beta}\bar{X}$$

$$= \hat{\beta}(X_t - \bar{X})$$

$$\Rightarrow \hat{y}_t = \hat{\beta}x_t \longrightarrow ⑧ \quad (\because X_t - \bar{X} = x_t)$$

Again, we have,

$$u_t = Y_t - \hat{Y}_t$$

$$= y_t - \hat{y}_t, \quad [\text{from } ⑥] \longrightarrow ⑨$$

Now, substituting ⑧ and ⑨ in $\Sigma\hat{y}_t u_t$, we get,

$$\Sigma\hat{y}_t u_t = \Sigma\hat{\beta}x_t(y_t - \hat{y}_t)$$

$$\Rightarrow \sum \hat{y}_t u_t = \sum \hat{\beta} x_t y_t - \sum \hat{\beta} x_t \hat{y}_t$$

$$= \hat{\beta} \sum x_t y_t - \hat{\beta} \sum x_t \hat{y}_t$$

$$= \hat{\beta} \sum x_t y_t - \hat{\beta} \sum x_t . \hat{\beta} \sum x_t$$

$$= \frac{\sum x_t y_t}{\sum x_t^2} . \sum x_t y_t - \hat{\beta}^2 \sum x_t^2$$

$$= \frac{(\sum x_t y_t)^2}{\sum x_t^2} - \frac{(\sum x_t y_t)^2}{(\sum x_t^2)^2} . \sum x_t^2$$

$$= \frac{(\sum x_t y_t)^2}{\sum x_t^2} - \frac{(\sum x_t y_t)^2}{\sum x_t^2}$$

$$= 0$$

Hence proved.

From eqn (7), we can get,

$$TSS = ESS + RSS \longrightarrow (10)$$

DERIVATION OF r^2 (R^2):

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we have,

$$\hat{y}_t = \hat{\beta} x_t \quad \text{[From eq}^n \text{ (8)]}$$

Now, squaring both sides and then taking summation we get,

$$\sum \hat{y}_t^2 = \hat{\beta}^2 \sum x_t^2$$

$$\Rightarrow \frac{\sum \hat{y}_t^2}{\sum y_t^2} = \frac{\hat{\beta}^2 \sum x_t^2}{\sum y_t^2} \text{, (Dividing both sides by } \sum y_t^2) \longrightarrow (11)$$

Since we have $\hat{\beta} = \frac{\sum x_t y_t}{\sum x_t^2}$, therefore from eqn (11),