Q 5(a). Check whether the following index numbers satisfy the time-reversal and factor reversal tests:

(i) $$I = \frac{\Sigma P_1 \sqrt{q_0 q_1}}{\Sigma P_0 \sqrt{q_0 q_1}} \times 100 \,;$$

(ii) $$I = \frac{\Sigma P_1 (q_0 + q_1)}{\Sigma P_0 (q_0 + q_1)} \times 100 \,;$$

(iii) $$I = \left(\frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \right)^{1/2} \times 100$$

Soln.: It is well-known to us that to satisfy the time reversal test, we must have,

$$P_{01} \times P_{10} = 1$$

and to satisfy the factor reversal test, we have to get,

$$P_{01} \times Q_{01} = V_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0}$$

(i) Here,

$$P_{01} = \frac{\Sigma P_1 \sqrt{q_0 q_1}}{\Sigma P_0 \sqrt{q_0 q_1}} \quad \text{(without the factor 100)}$$

This is nothing but the Walsch price I.N.

Now, $$P_{10} = \frac{\Sigma P_0 \sqrt{q_1 q_0}}{\Sigma P_1 \sqrt{q_1 q_0}}$$

$$\therefore \; P_{01}^{Wa} \times P_{10}^{Wa} = \frac{\Sigma P_1 \sqrt{q_0 q_1}}{\Sigma P_0 \sqrt{q_0 q_1}} \times \frac{\Sigma P_0 \sqrt{q_1 q_0}}{\Sigma P_1 \sqrt{q_1 q_0}} = 1$$

Hence Walsch price I.N. satisfies time reversal test.

Again $q_{01}^{wa} = \frac{\Sigma q_1 \sqrt{p_0 p_1}}{\Sigma q_0 \sqrt{p_0 p_1}}$

$$\therefore\ p_{01}^{wa} \times q_{01}^{wa} = \frac{\Sigma p_1 \sqrt{q_0 q_1}}{\Sigma p_0 \sqrt{q_0 q_1}} \times \frac{\Sigma q_1 \sqrt{p_0 p_1}}{\Sigma q_0 \sqrt{p_0 p_1}} \neq \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

Hence Walsch price index number does not satisfy the factor reversal test.

(ii)

$$P_{01} = \frac{\Sigma p_1 (q_0 + q_1)}{\Sigma p_0 (q_0 + q_1)} \quad \text{(without the factor 100)}$$

This is the Marshall - Edgeworth price Index Number

$$P_{10} = \frac{\Sigma p_0 (q_1 + q_0)}{\Sigma p_1 (q_1 + q_0)}$$

$$\therefore\ P_{01}^{ME} \times P_{10}^{ME} = \frac{\Sigma p_1 (q_0 + q_1)}{\Sigma p_0 (q_0 + q_1)} \times \frac{\Sigma p_0 (q_1 + q_0)}{\Sigma p_1 (q_1 + q_0)} = 1$$

Hence, Marshall - Edgeworth price index number satisfies time reversal test.

$$q_{01}^{ME} = \frac{\Sigma q_1 (p_0 + p_1)}{\Sigma q_0 (p_0 + p_1)}$$

$$\therefore\ P_{01}^{ME} \times q_{01}^{ME} = \frac{\Sigma p_1 (q_0 + q_1)}{\Sigma p_0 (q_0 + q_1)} \times \frac{\Sigma q_1 (p_0 + p_1)}{\Sigma q_0 (p_0 + p_1)} \neq \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

Thus, Marshall - Edgeworth index number does not satisfy factor reversal test.

(iii) $P_{01} = \left(\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \right)^{1/2}$ (without factor 100)

This given index number is nothing but Fisher's price index number.

$$P_{10} = \left(\frac{\Sigma P_0 q_1}{\Sigma P_1 q_1} \times \frac{\Sigma P_0 q_0}{\Sigma P_1 q_0} \right)^{1/2}$$

$$\therefore \quad P_{01}^F \times P_{10}^F = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times \frac{\Sigma P_0 q_1}{\Sigma P_1 q_1} \times \frac{\Sigma P_0 q_0}{\Sigma P_1 q_0}} = \sqrt{1} = 1$$

Hence, Fisher's index number satisfies time-reversal test.

Again,

$$q_{01}^F = \left(\frac{\Sigma q_1 P_0}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1} \right)^{1/2}$$

$$\therefore \quad P_{01}^F \times q_{01}^F = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}} \times \sqrt{\frac{\Sigma q_1 P_0}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}}$$

$$= \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times \frac{\Sigma P_0 q_1}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_1 q_0}}$$

$$= \sqrt{\frac{(\Sigma P_1 q_1)^2}{(\Sigma P_0 q_0)^2}} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0}$$

Hence, Fisher's price index satisfies time reversal test.