EX: Establish the relationship between Laspeyre's and Paache's price index number. when are the two equal ?

Solution:

We have, Laspeyre's price index number is given by $P_{01}^{La} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \rightarrow (1)$

[Base year method, Here base year quantities are takes as weights.]

and, Paache's price index is

$$P_{01}^{Pa} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \longrightarrow (2)$$

[Given are current year method. Here, given year quantities are taken as weights.]

To establish the relationship between Laspeyre's and Paache's index number, let us suppose that the following first.

X_j = Price relative $= \frac{p_1}{p_0}$

Y_j = quantity " $= \frac{q_1}{q_0}$

V_{01} = Value index no $= \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$

$W_j = p_0 q_0$, be the weights of X_j and Y_j $(j = 1, 2, \cdots n)$

r_{xy} = Correlation coefficient between (the price and quantity relatives) X and Y.

σ_x = Weighted S. D. of price - relative × [Dispersion of move' in price relative]

σ_y = " " " quantity - "

(5)

Now, we can get,

$$r_{xy} = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$$

$$\Rightarrow r_{xy}\,\sigma_x\sigma_y = Cov(X,Y)$$

$$= \frac{\sum w_j x_j y_j}{\sum w_j} - \left(\frac{\sum w_j x_j}{\sum w_j}\right)\left(\frac{\sum w_j y_j}{\sum w_j}\right)$$

$$= \frac{\sum p_1 q_1}{\sum p_0 q_0} - \frac{\sum p_1 q_0}{\sum p_0 q_0}\cdot\frac{\sum p_0 q_1}{\sum p_0 q_0}$$

$$= \frac{\sum p_1 q_1}{\sum p_0 q_0}\left[1 - \frac{\sum p_1 q_0}{\sum p_0 q_0}\cdot\frac{\sum p_0 q_1}{\sum p_1 q_1}\right]$$

$$\Rightarrow r_{xy}\,\sigma_x\sigma_y = V_{01}\left[1 - \frac{\sum p_1 q_0/\sum p_0 q_0}{\sum p_1 q_1/\sum p_0 q_1}\right]$$

$$\Rightarrow r_{xy}\,\sigma_x\sigma_y = V_{01}\left[1 - \frac{P_{01}^{La}}{P_{01}^{Pa}}\right]$$

$$\Rightarrow \frac{r_{xy}\,\sigma_x\sigma_y}{V_{01}} = 1 - \frac{P_{01}^{La}}{P_{10}^{Pa}}$$

$$\Rightarrow \frac{P_{01}^{La}}{P_{10}^{Pa}} = 1 - \frac{r_{xy}\,\sigma_x\sigma_y}{V_{01}} \longrightarrow (A)$$

Now from relation (A), we can infer three important relationships between Laspeyre's and Paasche's index number as follows –

<u>Case.1:</u> If $r_{xy} > 0$, then from (A) we get that R.H.S. is less than unity

$$\therefore \frac{P_{01}^{La}}{P_{01}^{Pa}} < 1 \quad (\text{L.H.S is also } <1)$$

$$\Rightarrow P_{01}^{La} < P_{01}^{Pa}$$

$$\frac{\frac{\sum p_1 q_0}{\sum p_0 q_0}\cdot\frac{\sum p_0 q_1}{\sum p_0 q_0}}{\frac{\sum p_1 q_1}{\sum p_0 q_0}}$$

$$= \frac{\sum p_1 q_0}{\sum p_0 q_0}\cdot\frac{\sum p_0 q_1}{\sum p_0 q_0}\cdot\frac{\sum}{\sum}$$

$$= \frac{\sum p_1 q_0}{\sum p_0 q_0}\cdot\frac{\sum p_0 q_1}{\sum p_1 q_1}$$

$n = \sum w_j$

$$\sum w_j x_j y_j = \sum p_0 q_0 \frac{p_1}{p_0}\cdot\frac{q_1}{q_0} = \sum p_1 q_1$$

$$\sum w_j x_j = \sum p_0 q_0 \frac{p_1}{p_0} = \sum p_1 q_0 = \sum p_1 q_0$$

$$\sum w_j y_j = \sum p_0 q_0 \frac{q_1}{q_0} = \sum p_0 q_1$$

$$Cov(X,Y) = E\left[\{X - E(X)\}\{Y - E(Y)\}\right]$$

$$= E(XY) - E(X)E(Y)$$

$$= \frac{1}{n}\sum x_i y_i - \left(\frac{1}{n}\sum x_i\right)\left(\frac{1}{n}\sum y_i\right)$$

$$= \frac{\sum w_j x_j y_j}{\sum w_j} - \left(\frac{\sum w_j x_j}{\sum w_j}\right)\left(\frac{\sum w_j y_j}{\sum w_j}\right)$$

Thus, when correlation coefficient between the price relatives X and the quantity relatives Y is greater than zero i.e., positive then Laspeyre's index is less than Paache's index number.

Case. II : If $r_{xy} < 0$, then in (A), we get that R.H.S. is greater than unity

$$\therefore \quad \frac{P_{01}^{La}}{P_{01}^{Pa}} > 1 \quad (\text{L.H.S. is also } <1)$$

$$\Rightarrow P_{01}^{La} > P_{01}^{Pa}$$

Thus, whenever correlation coefficient between X and Y is less than zero, then Laspeyre's index number is greater than Paache's index number.

The most important thing to be noted here is that, in actual practice, under normal economic conditions, we have $-1 \leq r_{xy} \leq 0$ and consequently we will get,

$$P_{01}^{La} > P_{01}^{Pa}$$

Thus, it is obvious that under normal conditions Laspeyre's price index number shows upward bias (i.e, Upper trend) while Paache's index number shows a downward bias (i.e, downward trend) in the price changes.

Case III: If either $r_{xy}=0$ or if $\sigma_x=0$, (i.e, all price movements are same for all commodities) or if $\sigma_y=0$ (i.e, all quantity movements are same for all commodities), then we get that R.H.S. of (A) is equal to unity.

$$\therefore \frac{P_{01}^{La}}{P_{01}^{Pa}}=1$$

$$\Rightarrow P_{01}^{La}=P_{01}^{Pa}$$

Thus, if or when either $r_{xy}=0$ or $\sigma_x=0$ or $\sigma_y=0$, then only Laspeyre's and Paache's index numbers become equal. That is to say, when either

(i) Correlation coefficient between price relatives x and quantity relatives y is zero.

or (ii) Weighted S.D. of price relatives of x is zero,

or (iii) Weighted S.D. of quantity relatives y is zero,

Then only it is possible that Laspeyre's and Paache's index number are equal.

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