EX: "Under normal circumstances, Laspeyre's price index number will show an upward bias and Paasche's a downward one of the same relative magnitudes in relation to time reversal test (TRT) and factor reversal test (FRT)." Explain.

Solution: To show that Laspeyre's price index number will show an upward bias in relation to TRT:

We know that,

$$\frac{P_{01}^{La}}{P_{01}^{Pa}} = 1 - \frac{r_{xy}\sigma_x\sigma_y}{V_{01}} \longrightarrow (1)$$

where, $r_{xy} = \frac{\text{cov}(x,y)}{\sigma_x\sigma_y}$ with usual notations.

Now, relation (1) can be written as –

$$\frac{\dfrac{\sum p_1q_0}{\sum p_0q_0}}{\dfrac{\sum p_1q_1}{\sum p_0q_1}} = 1 - \frac{r_{xy}\sigma_x\sigma_y}{V_{01}}$$

$$\Rightarrow \frac{\sum p_1q_0}{\sum p_0q_0} \cdot \frac{\sum p_0q_1}{\sum p_1q_1} = 1 - \frac{\sigma_x\sigma_y r_{xy}}{V_{01}} \longrightarrow (A)$$

$$\Rightarrow P_{01}^{La} \cdot P_{10}^{La} = 1 - \frac{r_{xy}\sigma_x\sigma_y}{V_{01}} \longrightarrow (2)$$

It is known to us that if Laspeyre's index number is to satisfy TRT, then we must get $P_{01}^{La} \cdot P_{10}^{La} = 1$.

But from (2), it is obvious that $P_{01}^{La} \cdot P_{10}^{La} = 1$, iff $r_{xy} = 0$, $\sigma_x = 0$, $\sigma_y = 0$ and then only we can say that TRT will be satisfied by Laspeyre's index number.

But under normal economic condition, we have $r_{xy} < 0$ i.e., the correlation coefficient between quantity relatives and price relatives is negative (and $\sigma_x \neq 0$, $\sigma_y \neq 0$), therefore right hand side of ② is greater than unity.

Hence, $P_{01}^{La} \cdot P_{10}^{La} > 1 \longrightarrow$ (B)

Thus it is obvious that under normal circum-stances Laspeyre's index number will shown an upward bias in relation to the time reversal test.

<u>To show that Paasche's index number will show an downward bias in relation to TRT</u>

we can have,

$$\frac{1}{P_{01}^{Pa} \cdot P_{10}^{Pa}} = \frac{1}{\frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \cdot \frac{\Sigma P_0 q_0}{\Sigma P_1 q_0}}$$

$$= \frac{\Sigma P_0 q_1}{\Sigma P_1 q_1} \cdot \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0}$$

$$\Rightarrow \frac{1}{P_{01}^{Pa} \; P_{10}^{Pa}} = P_{10}^{La} \cdot P_{01}^{La} \longrightarrow ③$$

Now, from (A) and ③, we get,

$$\frac{1}{P_{01}^{Pa} \; P_{10}^{Pa}} = 1 - \frac{r_{xy} \sigma_x \sigma_y}{V_{01}} \longrightarrow ④$$

∵ under normal circumstances, $r_{xy} < 0$, i.e, correlation coefficient between price relative and quantity relative is negative, there-fore, R.H.S. of (4) is greater than unity.

$$\therefore \frac{1}{P_{01}^{Pa} \cdot P_{10}^{Pa}} > 1$$

$$\Rightarrow P_{01}^{Pa} \cdot P_{10}^{Pa} < 1 \longrightarrow (5)$$

Thus, it is obvious that under normal condition, Paache's index number will show an downward bias in relation to TRT.

To show that Laspeyre's index number will show an upward bias in relation to FRT.

It is known to us that if Laspeyre's index number is to satisfy the FRT, then we must get,

$$P_{01}^{la} \cdot Q_{01}^{la} = V_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0}$$

Now,
$$P_{01}^{la} \cdot Q_{01}^{la} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \cdot \frac{\Sigma q_1 P_0}{\Sigma q_0 P_0}$$

$$= \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \cdot \frac{\Sigma P_0 q_1}{\Sigma P_0 q_0}$$

$$= \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \cdot \frac{\Sigma P_0 q_1}{\Sigma P_0 q_0} \cdot \frac{\Sigma P_1 q_1}{\Sigma P_1 q_1}, \left[\text{multiplying by } \frac{\Sigma P_1 q_1}{\Sigma P_1 q_1}\right]$$

$$= \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0} \cdot \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \cdot \frac{\Sigma P_0 q_1}{\Sigma P_1 q_1}$$

$$= V_{01} \left(P_{01}^{la} \cdot P_{10}^{la}\right)$$

$$\Rightarrow P_{01}^{la} Q_{01}^{la} = V_{01} \left(P_{01}^{la} \cdot P_{10}^{la}\right) > V_{01} \longrightarrow (6)$$

(4)

Sine $P_{01}^{da} \cdot P_{10}^{da} > 1$, already proved in (B). Thus, it is obvious that Laspeyre's index number shows an upward beas under normal condition in relation to FRT also as

$$P_{01}^{da} \cdot Q_{01}^{da} \neq V_{01}, \text{ but } P_{01}^{da} \cdot Q_{01}^{da} > V_{01} \text{ in } (6)$$

<u>To show that Paache's index number will show a downward beas in relation to FRT:</u>

Here,

$$P_{01}^{Pa} \cdot Q_{01}^{Pa} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \cdot \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}$$

$$= \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \cdot \frac{\Sigma P_1 q_1}{\Sigma P_1 q_0} \cdot \frac{\Sigma P_0 q_0}{\Sigma P_0 q_0} \quad \left[\text{multiplying by } \frac{\Sigma P_0 q_0}{\Sigma P_0 q_0}\right.$$

$$= \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0} \cdot \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \cdot \frac{\Sigma P_0 q_0}{\Sigma P_1 q_0}$$

$$= V_{01} \left(\frac{\Sigma P_1 q_1 / \Sigma P_0 q_1}{\Sigma P_1 q_0 / \Sigma P_0 q_0} \right)$$

$$= V_{01} \frac{P_{01}^{Pa}}{P_{01}^{da}} < V_{01} \longrightarrow (7)$$

Since $\frac{P_{01}^{Pa}}{P_{01}^{da}} < 1$, under normal condition.

Thus it is obvious that Paache's index number show a downward beas under normal condition in relation to FRT also as

$\frac{P_{01}^{Pa}}{P_{01}^{da}} < 1$ and hence $P_{01}^{Pa} \, Q_{01}^{Pa} \neq V_{01}$

rather $P_{01}^{Pa} \cdot Q_{01}^{Pa} < 1$ in (7).