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TEST OF ADEQUACY

Several formulae have been suggested for constructing index numbers and the problem is that of selecting the most appropriate one in a given situation. Therefore to choose an appropriate index, the following tests are suggested

1. Unit test
2. Time reversal test
3. Factor reversal test
4. Circular test

1. Unit test:

The unit test requires that the formula for constructing an index should be independent of the units in which, or for which, price and quantities are quoted. Except for the simple (unweighted) aggregative index all other formulae satisfy this test.

2. Time Reversal Test:

Prof. Irving Fisher has made a careful study of the various proposals for computing index numbers and has suggested various tests to be applied to any formulae to indicate whether or not it is satisfactory. The two most important of these he calls the time reversal test and factor reversal test.

Time reversal test is a test to determine whether a given method will work both ways in time, forward and backward. In the words of Fisher, "The test is that the formula for calculating the index number the should be such that it will give the same ratio between one point of comparision and the other, no matter which of the two is taken as base." In other words, when the data for any two years are treated by the same method, but with the bases reversal, the two index numbers secured should be reciprocal of each other so that their product is unity. Symbolically, the following relation should be satisfied :

$$P_{01} \times P_{10} = 1$$

where P_{01} is the index for time '1' on time '0' as base and P_{10} is the index for time '0' on time '1' as base. If the product is not unity, there is said to be a time bias in the method.

Thus, if from 1984 to 1985 the price of wheat increase from Rs. 120 to Rs. 140 per quintal the price in 1985 should be $133\frac{1}{3}$ percent of the prices in 1984 and the price in 1984 should be 75 percent of the price in 1985. One figure is the reciprocal of the other, their product (1.333×0.75) is unity. This is obviously true for each individual price relative and, according to the time reversal test, it should be true for the index number.

The test is not satisfied by Laspeyres method and the Paasches method as can be seen below:

Laspeyres method:

$$P_{01} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0}$$

$$\therefore \quad P_{10} = \frac{\Sigma P_0 q_1}{\Sigma P_1 q_1};$$

$$P_{01} \times P_{10} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_0 q_1}{\Sigma P_1 q_1} \neq 1$$

Thus, the test is not satisfied.

Paasches method:

$$P_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}, \quad P_{10} = \frac{\Sigma P_0 q_0}{\Sigma P_1 q_0}$$

$$\therefore \quad P_{01} \times P_{10} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times \frac{\Sigma P_0 q_0}{\Sigma P_1 q_0} \neq 1$$

Hence, the test is not satisfied.

There are five methods which do satisfy the test:

1. The Fisher's ideal formula
2. Simple geometric mean of price relatives.
3. Aggregates with fixed weights
4. Weighted geometric mean of price relatives if we use fixed weights.
5. Marshall Edgeworth method.

Fisher's ideal index number

Proof:
$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$$

changing time i.e, 0 to 1 and 1 to 0,

$$P_{10} = \sqrt{\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}}$$

$$= \sqrt{1}$$

$$= 1$$

Since $P_{01} \times P_{10} = 1$, the Fisher's ideal index satisfy the test.

3. Factor Reversal Test:

Another test suggested by Fisher's is known as Factor Reversal Test. It holds that the product of a price index and the quantity index should be equal to the corresponding value index. According to Fisher, "Just as each formula should permit giving the interchange of the two times without giving inconsistent results, so it ought to permit interchanging the prices and quantities without giving inconsistent result, i.e, the two results multiplied together should give the true value ratio." In other words, the test is that the change in price multiplied by the change in quantity should be equal to the total change in value. The total value of a given commodity in a given year is the product of the quantity and the price per unit (total value = $p \times q$). The ratio of the total value in one year to the total value in the preceding year is $\frac{p_1 q_1}{p_0 q_0}$.

If from one year to the next, both price and quantity should double the price relative would be 200, the quantity relative 200, and the value relative 400. The total value in the second year would be four times the value in the first year. In other words, if p_1 and p_0 represented prices and q_1 and q_0 the quantities in the current year and the base year respectively. and if P_{01} represents the change in price in the current year and Q_{01} the change in quantity in the current year, then

$$P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

If the product is not equal to the value ratio, there is, with reference to this test, an error in one or both of the index numbers.

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The factor reversal test is satisfied only by the Fisher's ideal index.

Proof:

$$P_{01} = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}}$$

changing p to q and q to p,

$$Q_{01} = \sqrt{\frac{\Sigma q_1 P_0}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}}$$

Now,

$$P_{01} \times Q_{01} = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times \frac{\Sigma q_1 P_0}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}}$$

$$= \sqrt{\frac{(\Sigma P_1 q_1)^2}{(\Sigma P_0 q_0)^2}}$$

$$= \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0}$$

Since $P_{01} \times Q_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0}$, the factor reversal test is satisfied by the Fisher's Ideal Index.

This means, of course, that the formula serves equally well for constructing indices of quantities as for constructing indices of prices, the quantity index being derived by interchanging p and q in the ideal formula.

4. Circular Test :

Another test of the adequacy of index number formula is what is known as 'Circular test'. If in the use of index numbers interest attaches not merely to a comparision of two years, but to the measurement of price changes over a period of years, it is fequently desirable to shift the base. A formula is said to meet this test, if, for eg. the 1985 index with 1980 as the base is 200, and the 1980 index with ~~with~~ 1975, as the base is again 200; then the 1985 index with 1975 as the base must be 400. Clearly, the desirability of this property is that it enables us to adjust the index number values from period to period without refering each time to the original base. A test of this shiftability of base is called the circular test.

This test is just an extension of the time reversal test. The test requires that if an index is constructed for the year on a base year b, and for the year b on base year C, we ought to get the same result as if calculated direct an index for A on base year C withought going through b as an intermediary.

Symbolically, if there are three indices

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P_{01}, P_{12} and P_{20}, the circular test will be satisfied if $P_{01} \times P_{12} \times P_{20} = 1$

The Laspeyre's index does not satisfy the test as can be seen from the following

If the three years are 0, 1, 2, the index being Laspeyre's method will be

$$\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_2 q_1}{\Sigma P_1 q_1} \times \frac{\Sigma P_0 q_2}{\Sigma P_2 q_2}$$

The product of all these is not equal to one. Hence, the test is not satisfied. Similarly, it can be shown that the Paaches index and Fisher index do not satisfy the test. However the simple aggregative method and the fixed weight aggregative method satisfy the test as can be seen from the following –

When the test is applied to the simple aggregative method, we will get,

$$\frac{\Sigma P_1}{\Sigma P_0} \times \frac{\Sigma P_2}{\Sigma P_1} \times \frac{\Sigma P_0}{\Sigma P_2} = 1$$

Similarly, when applied to fixed weight aggregative method, we will get,

$$\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_2 q}{\Sigma P_1 q} \times \frac{\Sigma P_0 q}{\Sigma P_2 q} = 1$$

An index number which satisfies this test has the advantage of reducing the computation every time a change in the base year has to be made. Such index numbers can be adjusted from year to year without referring each time to the original bases.

The circular test is not met by the ideal index or by any of the weighted aggregatives with changing weights. The test is met by simple geometric mean of price relatives and the aggregatives fixed weights. The reason why the Laspeyre's and Paaches index numbers and their derivatives, the Marshall - Edgeworth and the ideal indices do not meet the circular test is that the weights in these index numbers depend on the periods between which comparisions are being made. If these periods change the weights change. For example, if the base period is taken as period 2 rather from period 0, the weights to Laspeyre's index are no larger q_0 but q_2.