Ex. Find the root of the equation

$x \log_{10} x - 1.2 = 0$, correct to five decimal places,

by Regula-Falsi method.

Soln. Given eqn. is

$$x \log_{10} x - 1.2 = 0 \longrightarrow (1)$$

Let,

$$y = f(x) = x \log_{10} x - 1.2.$$

Then we have

$$f(2) = 2 \log_{10} 2 - 1.2 = 2 \times (.30103) - 1.2 = -0.59794$$

$$f(3) = 3 \log_{10} 3 - 1.2 = 3 \times (.47712) - 1.2 = 0.23136.$$

$\because$ $f(2)$ & $f(3)$ are of opposite signs, hence there exists at least one real root of the eqn. $f(x) = 0$ in the interval $(2, 3)$.

$\therefore$ For the first approximation, we get-

$$x_1 = 2, \quad y_1 = -0.59794$$

$$x_2 = 3, \quad y_2 = 0.23136.$$

& first approximation of the root is

$$x^{(1)} = x_1 + \frac{|x_2 - x_1|\,|y_1|}{|y_1| + |y_2|}$$

$$= 2 + \frac{|3-2| \times (0.59794)}{0.59794 + 0.23136} = 2 + \frac{0.59794}{0.8293}$$

$$= 2.721$$

Also, we have

$$f(2.721) = 2.721 \times \log_{10} 2.721 - 1.2$$
$$= 2.721 \times 0.43473 - 1.2$$
$$= 1.1829 - 1.2$$
$$= -0.0171.$$

Further, $f(2.721)$ and $f(3)$ are of opposite signs; so root lies in the interval $(2.721, 3)$.

$\therefore$ For 2nd approximation, we take

$$x_1 = 2.721, \quad y_1 = -0.0171$$
$$x_2 = 3, \quad y_2 = 0.23136$$

& The 2nd approximation of the root is

$$x^{(2)} = x_1 + \frac{|(x_2 - x_1)|\,|y_1|}{|y_1| + |y_2|}$$
$$= 2.721 + \frac{|(3 - 2.721)| \times (0.0171)}{(0.0171) + (0.23136)}$$
$$= 2.721 + \frac{0.06477}{0.24846}$$
$$= 2.721 + 0.019198$$
$$= 2.740198$$
$$= 2.74020$$

Also,
$$f(2.74020) = 2.7402 \times \log_{10}(2.7402) - 1.2$$
$$= 2.7402 \times (0.43778) - 1.2$$
$$= -0.0003952 = -0.0004$$

Again as $f(2.740200)$ and $f(3)$ are of opposite signs; hence the root lies in the interval $(2.7402, 3)$.

$\therefore$ For the 3rd approximation we have

$$x_1 = 2.7402, \quad y_1 = -0.0004$$

$$x_2 = 3, \quad y_2 = 0.23136$$

& the 3rd approximation of the root is

$$x^{(3)} = x_1 + \frac{|(x_2 - x_1)|\,|y_1|}{|y_1| + |y_2|}$$

$$= 2.7402 + \frac{|3 - 2.7402| \times (0.0004)}{0.0004 + 0.23136}$$

$$= 2.7402 + \frac{0.00010392}{0.23176}$$

$$= 2.7402 + 0.00044839$$

$$= 2.74064839$$

$$\cong 2.74065$$

Hence The required root is 2.74065.

$\times$