Q. Prove that the magnetic moment of an electron is given by $\frac{\mu e}{2}$ $P_m = -\frac{e}{2m}\vec{L}$ where e = charge of electron, m = electron mass and L = orbital angular momentum due to orbital motion. Hence show that magnetic moment due to orbital motion of an electron must be an integral multiple of $\frac{e\hbar}{2m}$.

How do you define a Bohr magneton. Give it's numerical value. G.U - 2000.

Ans— Magnetic moment of an electron— All substances are composed of atoms and an atom consist of a central positively charged nucleus where whole mass of the atom is supposed to be concentrated, with suitable number of electrons ~~revolv~~ revolving round it in more or less circular orbits. The revolution of an electron in a clockwise direction is equivalent to a conventional current in the anticlockwise direction and the electronic orbit behaves like a magnetic dipole.

Consider the simplest atom of hydrogen. It consist of one proton in the nucleus and one electron revolving round it. If m = mass of the electron, $-e$ = charge on it v = its velocity and r = radius of the orbit. Then the centripetal force acting on the electron is

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e.e}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

$$\text{or } v = \left[\frac{1}{4\pi\epsilon_0}\frac{e^2}{mr}\right]^{1/2}$$

Now $m = 9\times10^{-31}$ kg, $e = 1.6\times10^{-19}$ C for hydrogen

$r = 0.5\times10^{-10}$ m, $\frac{1}{4\pi\epsilon_0} = 9\times10^9$ Nm^2/C^2

$$\therefore v = \left[\frac{9\times10^9\times1.6\times1.6\times10^{-38}}{9\times10^{-31}\times0.5\times10^{-10}}\right]^{1/2} = 2.262\times10^6 \text{ m/sec.}$$

and time taken by one electron to complete one rotation

$$T = \frac{2\pi r}{v} = \frac{2\pi\times0.5\times10^{-10}}{2.262\times10^6} = 1.39\times10^{-16} \text{ sec.}$$

Current in the orbit — From the above calculation we find that the time taken by the electron to complete one orbit is very very small. The current i in the orbit is given by

$$i = \frac{\text{charge}}{\text{Time taken to complete one rotation}} = \frac{e}{2\pi r/v}$$

$$= \frac{ev}{2\pi r}$$

Orbital magnetic moment:- The magnitude of the dipole magnetic moment of the current loop is given by

$$P_m = iA = \frac{ev}{2\pi r}\pi r^2 = \frac{1}{2}evr.$$

The magnitude of angular momentum of the electron revolving round the nucleus $L = mvr$.

$$\therefore P_m = \frac{1}{2}\frac{e}{m}L. \longrightarrow (1)$$

As $\vec{P}_m$ and $\vec{L}$ are both vector quantities and electron charge e is negative, eqn (1) can be put as $\vec{P}_m = -\frac{e}{2m}\vec{L} \longrightarrow (ii)$

According to Bohr theory an electron can only revolve in an orbit in which it's total angular momentum is an integral multiple of $\frac{h}{2\pi}$ where h is plank's constant.

$$L = n\frac{h}{2\pi} \quad \text{where } n \text{ is an integer.}$$

Putting the value of L in (1) we have

$$P_m = \frac{1}{2}\frac{e}{m}\frac{nh}{2\pi} = \frac{neh}{4\pi m}$$

$$\text{or } P_m = \frac{ne\hbar}{2m} \longrightarrow (iii) \quad \left(\hbar = \frac{h}{2\pi}\right)$$

From (iii) we find that the magnetic moment due to orbital motion of an electron must be an integral multiple of $\frac{e\hbar}{2\pi}$.

Bohr magneton :- The smallest value of orbital magnetic moment of the electron for $n=1$ is given by $P_m = \frac{e\hbar}{2m} = \frac{eh}{4\pi m}$

$$\longrightarrow (iv)$$

This is the unit of magnetic moment and is known as Bohr magneton. It is denoted by μ_B.

<u>Numerical value of μ_B</u> :- we have $e = 1.6 \times 10^{-19}$ c.

$h = 6.6 \times 10^{-34}$ Joul sec

$m = 9 \times 10^{-31}$ kg.

$$\therefore \mu_B = \frac{eh}{4\pi m} = \frac{1.6 \times 10^{-19} \times 6.6 \times 10^{-34}}{4\pi \times 9 \times 10^{-31}}$$

$$= 9.27 \times 10^{-24} \text{ Joul/Tesla}$$