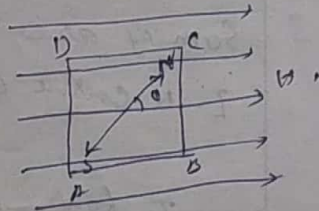


Hysteresis loss

When magnetic substance is subjected to a magnetising field, the magnetic di-poles of the substance experience torque which tends to rotate them to get them oriented in the direction of the field. ~~To zero~~, to achieve this some energy has to be spent. On reducing the field to zero, some residual magnetism is retained by the substance. So the total energy spent is not recovered. As such there is some loss of energy in this process. This energy is lost in the form of heat produced in the sample.

The loss of energy which takes place when a magnetic substance is taken over a complete cycle of magnetisation is called hysteresis loss.

Consider a unit volume ABCD of the material subjected to a magnetising field. Let one of ~~the~~ its magnetic di-poles of magnetic moment M be inclined at an angle θ with the direction of field.



Component of magnetic moment of the di-pole parallel to the field = $M \cos \theta$

Since sum of the components of magnetic moments per unit volume parallel to the field is called the intensity of magnetisation $I = \sum M \cos \theta$

$$\text{Differentiating w.r.t } \theta \quad dI = -\sum M \sin \theta d\theta \quad \text{--- (1)}$$

Torque τ acting on the di-pole is

$$\tau = MH \sin \theta d\theta$$

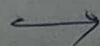
Work done in rotating the di-pole through an angle $d\theta$
 $= MH \sin \theta (-d\theta)$

The $-ve$ sign is due to the fact the field tends to decrease the angle.

\therefore work done in rotating all the dipoles in unit volume through an angle $d\theta = -\sum MH \sin \theta d\theta$

$$= -H \sum M \sin \theta d\theta$$

$$= H dI \quad \text{[using (1)]}$$



Net work done in changing the intensity of magnetisation of the unit volume of sample from I_1 to I_2

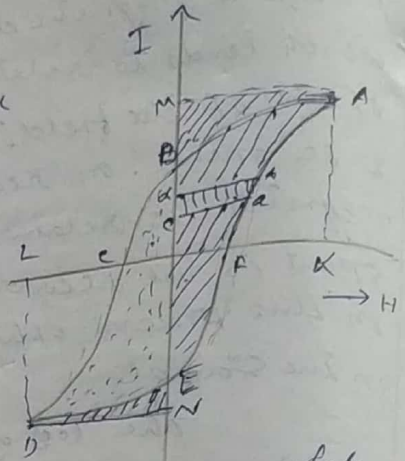
$$= \int_{I_1}^{I_2} H dI$$

Let a and b be two points situated very close to each other on part FA of the hysteresis loop $ABCDEFA$

$$ac \approx bd \approx H \text{ and}$$

$$cd = dI$$

$$\therefore H dI = a c \times c d = \text{area of strip } ABCD.$$



Therefore $\int H dI$ represents the sum of areas of these type of strips in which the whole $I-H$ curve can be divided, whole cycle of magnetisation can be divided into four parts

(i) When field increases from 0 to K , work done on the sample

$$W_1 = \int_E^M H dI = \text{Area of } EFAMBE$$

(ii) When field decreases from K to zero, work done W_2 by the sample $W_2 = \int_M^B H dI = \text{Area } AMBA$

(iii) When field is increased from zero to L in opposite direction work done W_3 on the sample is

$$W_3 = \int_B^N H dI = \text{Area } BCDNEB$$

(iv) When field is decreased from L to zero, work done W_4 by the sample $W_4 = - \int_N^E H dI = - \text{Area } DNED$

If W is the net work done on the sample during one complete cycle of magnetisation

$$W = W_1 + W_2 + W_3 + W_4$$

$$W = \text{Area EFAMOE} - \text{Area AMBA} + \text{Area BCDNEB} - \text{Area DNED}$$

$$= \text{Area ABCDEFA}$$

= Area enclosed by I-H loop.

Thus the loss of energy per unit volume, when the sample is taken through a complete cycle of magnetisation, is equal to the area enclosed by I-H loop.

$$\text{we can write } W = \oint H dI \quad \text{--- (ii)}$$

the \oint denotes that the integration is carried around a complete cycle. Eqn (ii) is called Wahburg's law.

Hysteresis loss in terms of B-H curve

magnetic induction B is given by

$$B = H + 4\pi I$$

Differentiating both sides

$$dB = dH + 4\pi dI$$

Multiplying both sides by H and performing the integration over a complete cycle.

$$\oint H dB = \oint H dH + 4\pi \oint H dI$$

Since a graph of H plotted against I is a straight line

$$\therefore \oint H dH = 0$$

$$\therefore \oint H dB = 4\pi \oint H dI$$

or Area of B-H curve = $4\pi \times$ Area of I-H curve.

Area of B-H curve = $4\pi \times$ (loss of energy per unit volume)

$$\therefore \text{Loss of energy per unit volume} = \frac{1}{4\pi} (\text{area of B-H curve})$$

Langevin's Theory of Paramagnetism

Langevin gave a theoretical interpretation of paramagnetism on the basis of the kinetic theory of gases and obtained a quantitative relation for the paramagnetic gas which justify the experimental results of Curie's law.

The molecules of the paramagnetic gas are supposed to be small permanent magnets due to circulating electrons. When the external magnetic field is not present, the magnetic axes of the molecules are uniformly distributed in all directions. Hence the sum of the projections of the elementary moments on any line of reference is zero. When the magnetic field is applied the molecules will tend to orient themselves with their magnetic axes in the field direction.

Let us consider the unit volume of paramagnetic gas containing n molecules each bearing a permanent magnetic moment M . If the magnetic field H is applied, the potential energy of the molecules, whose magnetic axis makes an angle θ with the magnetic field H , is $-MH \cos \theta$.

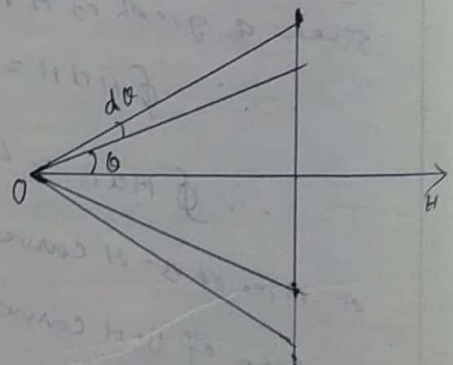
The total magnetic moment per unit volume in the direction of field $M = nM \overline{\cos \theta}$ ——— (1)

where $\overline{\cos \theta}$ = Average value of $\cos \theta$

n = No. of molecules per unit volume

M = Permanent dipole moment

Let us assume that all the molecules of the gas have been collected and placed with their centres at point O and their axes pointing in the same directions in which they actually pointed as they were collected.



The no. of molecules whose axes fall within the solid angle between two hollow cones of semi angles θ and $\theta + d\theta$ is given by

$$dn = 2\pi \sin \theta d\theta e^{MH \cos \theta / KT} \quad \text{--- (2)}$$

where $2\pi \sin \theta d\theta$ = Solid angle between θ and $\theta + d\theta$.

K = Boltzmann Constant.

The average value of magnetic moment in the field direction is

$$\overline{\mu \cos \theta} = \frac{\int_0^\pi \mu \cos \theta d\theta}{\int_0^\pi d\theta} \quad \text{--- (3)}$$

$$\begin{aligned} \overline{\mu \cos \theta} &= \frac{\int_0^\pi \mu \cos \theta \cdot 2\pi \sin \theta d\theta \cdot e^{\mu H \cos \theta / kT}}{\int_0^\pi 2\pi \sin \theta d\theta \cdot e^{\mu H \cos \theta / kT}} \\ &= \frac{\mu \int_0^\pi \cos \theta \sin \theta d\theta \cdot e^{\mu H \cos \theta / kT}}{\int_0^\pi \sin \theta d\theta \cdot e^{\mu H \cos \theta / kT}} \quad \text{--- (4)} \end{aligned}$$

$$\text{Let } x = \frac{\mu H \cos \theta}{kT} \text{ and } \frac{\mu H}{kT} = a \quad \text{--- (5)}$$

$$\Rightarrow \cos \theta = \frac{kT x}{\mu H} \quad \therefore -\sin \theta = \frac{kT dx}{\mu H}$$

$$\text{When } \theta = 0, x = \frac{\mu H}{kT} = +a$$

$$\text{When } \theta = \pi, x = -\frac{\mu H}{kT} = -a$$

Putting these values in (4), we get

$$\overline{\mu \cos \theta} = \frac{\mu \int_{-a}^{+a} \frac{kT}{\mu H} x \left(-\frac{kT dx}{\mu H} \right) e^x}{\int_{-a}^{+a} \left(-\frac{kT dx}{\mu H} \right) e^x} \quad \text{--- (6)}$$

$$= \frac{\mu \times \frac{1}{a} \int_{-a}^{+a} x e^x dx}{\int_{-a}^{+a} e^x dx} \quad \left[\because \int_{-a}^{+a} -dx = \int_{-a}^{+a} dx \right]$$

$$= \frac{\mu \times \frac{1}{a} \left[x e^x - e^x \right]_{-a}^{+a}}{\left[e^x \right]_{-a}^{+a}}$$

$$\therefore \overline{\mu \cos \theta} = \frac{\mu}{a} \left[\frac{(a e^a + a e^{-a}) - (e^a - e^{-a})}{(e^a - e^{-a})} \right]$$

$$= \frac{\mu}{a} \left[\frac{a(e^a + e^{-a})}{(e^a - e^{-a})} - 1 \right]$$

$$= \mu \left[\frac{e^a + e^{-a}}{e^a - e^{-a}} - \frac{1}{a} \right]$$

$$\mu \overline{\cos \theta} = \mu \left[\cos \theta_0 - \frac{1}{2} \right] \text{ where } \cos \theta_0 = \frac{v_{rel}^2}{c^2}$$

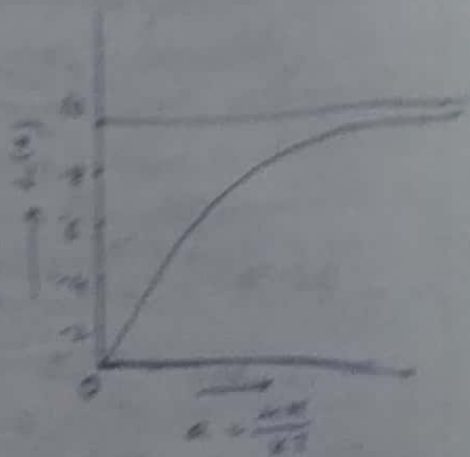
$$\mu \overline{\cos \theta} = \mu L(x) \quad \text{--- (1)}$$

where $L(x) = \cos \theta_0 - \frac{1}{2} = \text{Langevin's function}$

Now a graph is plotted betⁿ $L(x)$

and x as shown in the fig.

This curve is known as Langevin curve, for large values of x , i.e. for high field strengths and for low Temp^s $L(x)$ approaches unity only, when all μ are for low field strengths and for high



Temp^s $\frac{\mu H}{kT} \ll 1$ or $\mu H \ll kT$ then

$$L(x) = \frac{x}{3} - \frac{1}{3} \frac{\mu^2 H^2}{k^2 T^2} \quad \text{--- (2)}$$

Using (2) in (1), we get

$$\mu \overline{\cos \theta} = L(x) \frac{1}{3} \frac{\mu H}{kT}$$

$$\therefore \mu \overline{\cos \theta} = \frac{\mu^2 H}{3kT} \quad \text{--- (3)}$$

Using (3) in eqⁿ (1), we get

$$\mu = \frac{2\mu^2 H}{3kT}$$

Since the magnetic susceptibility

$$\chi = \frac{\mu}{H} = \frac{2\mu}{3kT} = \frac{C}{T} \quad \text{--- (4)}$$

$$\text{where } \frac{2\mu^2}{3k} = C$$

If we consider a gas molecule of the gas, we have

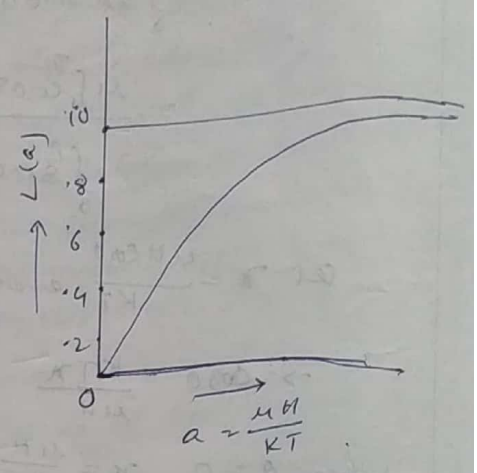
$$\chi = \frac{\mu}{H} = \frac{(2\mu^2)}{3kT} = \frac{C}{T} \quad \text{--- (5)}$$

$$\propto \overline{M \cos \theta} = \mu \left[\coth a - \frac{1}{a} \right] \text{ where } \coth a = \frac{e^a + e^{-a}}{e^a - e^{-a}}$$

$$\therefore \overline{M \cos \theta} = \mu L(a) \quad \text{--- (7)}$$

where $L(a) = \coth a - \frac{1}{a} = \text{Langevin's function}$.

Now a graph is plotted betⁿ $L(a)$ and 'a' as shown in the fig. This curve is known as Langevin curve. For large value of a, i.e. for high field strength and for low Temp $L(a)$ approaches unity. when $a \ll 1$ i.e. for low field strength and for high



Temp $\frac{\mu H}{KT} \ll 1$ or $\mu H \ll KT$ then

$$L(a) = \frac{a}{3} = \frac{1}{3} \frac{\mu H}{KT} \quad \text{--- (8)}$$

using (8) in (7), we get

$$\overline{M \cos \theta} = \mu \times \frac{1}{3} \frac{\mu H}{KT}$$

$$\overline{M \cos \theta} = \frac{\mu^2 H}{3KT} \quad \text{--- (9)}$$

Using (9) in eqn (1), we get

$$M = \frac{n \mu^2 H}{3KT}$$

Hence the magnetic susceptibility

$$\chi = \frac{M}{H} = \frac{n \mu^2}{3KT} = \frac{C}{T} \quad \text{--- (10)}$$

where $\frac{n \mu^2}{3K} = C$

If we consider a gram-molecule of the gas, the molar susceptibility

$$\chi_M = \frac{N \mu^2}{3KT} = \frac{(N \mu)^2}{3NKT} = \frac{\sigma_0^2}{3RT} = \frac{C_M}{T} \quad \text{--- (11)}$$

Bohr's Magnetism

Where $C_m = N\mu = \text{gram-molecular magnetic moment}$,

If $a \gg 1$ i.e. $\frac{4\pi M}{kT} \gg 1$ or $MH \gg kT$ then $L(a) = 1$

$$\therefore M = nM$$

The eqn (11) shows that the paramagnetic susceptibility χ_m varies inversely as absolute temperature T , which explains the result obtained by Curie on his extensive experimental research and is expressed as

$$\chi_m T = C_m \quad \text{--- (12)}$$

where $C_m = \text{Curie Constant, for gram-molecules}$