Bohr Magneton

As the electron in the hydrogen atom rotates around the positive nucleus, it constitute a circular current and thus atom requires a magnetic moment.

If e = charge of an electron.

T = Time to complete one full rotation in a circular orbit then

current $i = \frac{e}{T}$

Due to this circular current the magnetic moment of the atom = $i \times$ area of the orbit.

$= i \times \pi r^2$ where r = radius of the orbit.

$\therefore$ magnetic moment of the atom = ~~$\frac{e}{T} \times \pi r^2$~~

$= \frac{e}{T} \times \pi r^2$

$= \frac{e\omega}{2\pi} \times \pi r^2 \quad \left(T = \frac{2\pi}{\omega} \right)$

$= \frac{1}{2} e\omega r^2 \longrightarrow (1)$.

Where ω = Angular velocity of the electron in a circular orbit.

Since Angular momentum of an electron

$= mvr = mr\omega r \quad (\because v = r\omega)$

$= mr^2\omega$

$= \frac{nh}{2\pi}$ [according Bohr's theory of H-spectra]

$\therefore mr^2\omega = \frac{nh}{2\pi}$

$\therefore \omega = \frac{nh}{2\pi mr^2}$ ——— (1)

using equ^n (2) in (1), we get

magnetic moment of the atom

$= \frac{1}{2}er^2 \times \frac{nh}{2\pi mr^2}$

$= n\left(\frac{eh}{4\pi m}\right)$

Thus the magnetic moment of the atom is an integral multiple of $\left(\frac{eh}{4\pi m}\right)$, which is known as Bohr magneton.

If $\frac{e}{m} = 1.7589 \times 10^{11}$ C/kg.

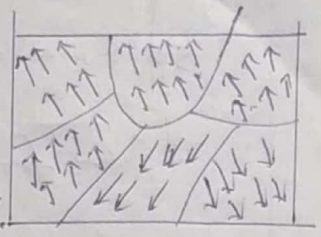
and $h = 6.62 \times 10^{-34}$ Joul-sec, then

$\frac{eh}{4\pi m} = \frac{1.7589 \times 10^{11} \times 6.62 \times 10^{-34}}{4\pi}$

$= 9.265 \times 10^{-24}$ A-m^2

Weiss Field Theory of Ferromagnetism.

Weiss theory of ferromagnetism is based on the following assumptions
(i) The atomic dipoles of the ferromagnetic substances are grouped together in domains as shown in the fig.



In each domain the dipoles are all aligned in the same direction and have magnetic moment. In absence of external magnetic field the domains is spontaneously magnetised. The direction of magnetisation in different domains are different in an unmagnetised substance. Hence on average the resultant magnetisation is zero.

(ii) The cause of spontaneous magnetisation within each domain is the existance of the molecular field which tries to produce a parallel alignment of the dipoles. Again, this internal field H_m varies as the magnetisation M within a domain.

Thus $H_m \propto M$ or $H_m = \gamma M$ ——— (1)

Where γ is the molecular field constant, γ does not depend upon Temp^r. H represent the external magnetic field, hence the effective magnetic field acting on an atom or ion can be written as $H_e = H + \gamma M$ ——— (2)

Let us assum a solid.

Let N = Number of atom per unit volume

J = Total angular momentum quantum number of each atom, then the possible components of magnetic moment is $M_J g \mu_B$ ——— (3)

Where $M_J = J, (J-1), \cdots, -(J-1) - J$

= Magnetic quantum number associated with J.

Thus the potential energy of atomic dipole with Component $M_J g \mu_B$ along $H = -M_J g \mu_B H$ ——— (4)

Where g = Lande's splitting factor

$$= 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

→

μ_B = Bohr-magneton

L = Orbital angular momentum of each atom

S = Spin angular momentum of each atom.

Now from the principle of statistical mechanics the total magnetic moment per unit volume or the magnetisation along H is

$$M = \frac{N \sum_{-J}^{+J} M_J g \mu_B e^{M_J g \mu_B H/KT}}{\sum_{-J}^{+J} e^{M_J g \mu_B H/KT}} \longrightarrow (5)$$

Solving the above eqn we get

$$M = N g J \mu_B B_J(x) \longrightarrow (6)$$

where $x = \frac{g J \mu_B H}{KT}$

$B_J(x)$ = Brillouin function.

For magnetic material H is replaced by $H + \gamma M$, hence we get $x = \frac{g J \mu_B (H + \gamma M)}{KT} \longrightarrow (7)$

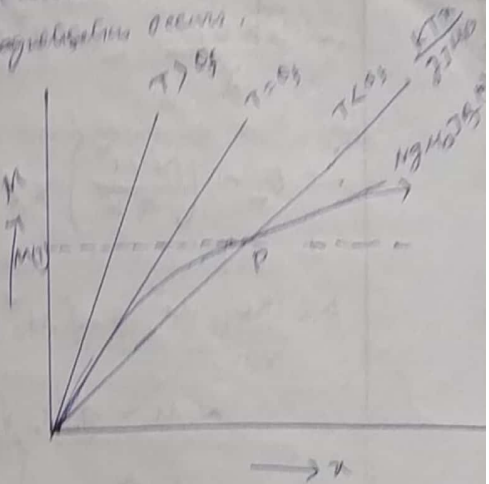
For spontaneous magnetisation, $H = 0$, hence ~~from~~ the form (7) becomes

$$x = \frac{g J \mu_B \gamma M}{KT}$$

$$\therefore M = \frac{KT x}{g J \mu_B \gamma} \longrightarrow (8)$$

A M/H graph is plotted between M and H. The eqn (8) represents a straight line whose slope is proportional to T. The graph is shown in the fig below.

For $T < \theta_f$ (Curie point) i.e temp at and above which spontaneous magnetisation vanishes) spontaneous magnetisation occurs.



For $T = \theta_f$, the st. line represented by (8) becomes tangent of the curve.

For $T > \theta_f$, the spontaneous magnetisation vanishes.

In order to establish a relation between the Curie temp θ_f and molecular field constant γ, we assume that $x << 1$ (near the origin), the Brillouin function is approximately given by

$$B_J(x) = \frac{(J+1)x}{3J} \longrightarrow (9)$$

using (9) in (6), we get

$$M = \frac{Ng J \mu_B (J+1) x}{3J}$$

$$\therefore M = \frac{Ng \mu_0 (J+1) x}{3} \longrightarrow (10)$$

The slope of this is

$$\frac{Ng \mu_0 (J+1)}{3} \longrightarrow (11)$$

This is the slope of the tangent of the curve at the origin putting $T = \theta_f$ in (8), we get

$$\text{slope} = \frac{k \theta_f}{g J \mu_0 \gamma} \longrightarrow (12)$$

From (11) and (12)

→

$$\frac{k\theta_f}{gJ\mu_B\gamma} = \frac{Ng\mu_B(J+1)}{3}$$

$$\text{or } \frac{3k\theta_f}{\gamma} = Ng^2\mu_B^2 J(J+1) \longrightarrow (13)$$

$$\therefore \theta_f = \left(\frac{N\mu^2}{3k}\right)\times\gamma \longrightarrow (14)$$

where $\mu^2 = g^2\mu_B^2 J(J+1)$

$$\text{or } \mu = g\mu_B\sqrt{J(J+1)}$$

Since $\frac{N\mu^2}{3k}$ is constant.

$$\therefore \theta_f \propto \gamma$$

Let us now assume susceptibility in the region well above the ferromagnetic curie Tempr. In this region magnetisation occurs only when external magnetic field H is applied. because there is no spontaneous magnetisation

From (10) $M = \frac{Ng\mu_B(J+1)\chi}{3}$

From (7) $\chi = \frac{gJ\mu_B(H+\gamma M)}{KT}$

$$\therefore M = \frac{Ng\mu_B(J+1)}{3}\times\frac{gJ\mu_B(H+\gamma M)}{KT}$$

$$\text{or } M = \frac{Ng^2\mu_B^2 J(J+1)(H+\gamma M)}{3KT}$$

$$\therefore M = \frac{N\mu^2(H+\gamma M)}{3KT}$$

where $\mu^2 = g^2\mu_B^2 J(J+1)$

$\rightarrow$

$$\frac{M}{H} = \frac{N\mu^2}{3KT}\left(1+\gamma\frac{M}{H}\right)$$

$$\text{or } \chi = \frac{N\mu^2}{3KT}(1+\gamma\chi)$$

where $\chi = \frac{M}{H}$ = Susceptibility

$$\text{or } \chi\left[1-\gamma\frac{N\mu^2}{3KT}\right] = \frac{N\mu^2}{3KT}$$

$$\text{or } \chi\left(1-\frac{\theta}{T}\right) = \frac{C}{T} \quad \text{where } \theta = \frac{\gamma N\mu^2}{3K} \text{ and.}$$

$$C = \frac{N\mu^2}{3K}.$$

$$\therefore \chi = \frac{\frac{C}{T}}{1-\theta/T} = \frac{C}{T\left(1-\frac{\theta}{T}\right)}$$

$$\therefore \chi = \frac{C}{T-\theta} \longrightarrow (16)$$

Thus we observe that the value of θ obtained from (16) is identical with that obtained for θ_f from (14). Hence we can say that weiss theory does not distinguish the para and ferromagnetic Curie temp^{re}.