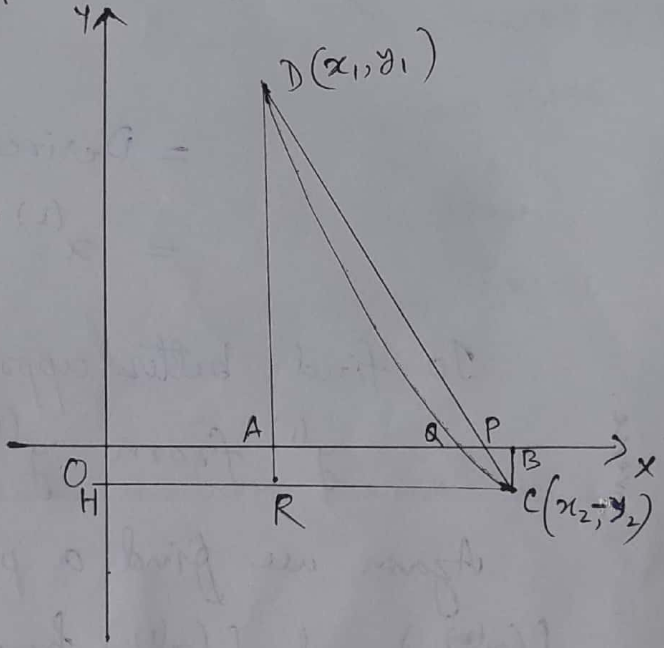
(2) Regula-Falsi method or method of false position:

To find a real root of $f(x) = 0$, we find a sufficiently small interval (x_1, x_2) in which the root of the eqn. lies.

Therefore, the graph of the fn. $y = f(x)$ will surely cross the x-axis between $x = x_1$ and $x = x_2$ which shows that $f(x_1)$ and $f(x_2)$ are of opposite signs.

Now, as the interval (x_1, y_1) is sufficiently small, the curve of the function in this interval may be considered as a straight line.



From the adjacent figure $OP = x$ is the approximate value of the root, while Oa is the actual value of the root. Now, we have

$$OP = OA + AP$$

$$= x_1 + AP \longrightarrow (1)$$

Now, triangle DAP and DRC are similar.

$$\therefore \frac{DA}{DR} = \frac{AP}{RC}$$

$$\Rightarrow AP = \frac{RC}{DR} \cdot DA$$

$$= \frac{(x_2 - x_1)}{DA + AR} \cdot DA$$

$$= \frac{(x_2 - x_1)\,|y_1|}{|y_1| + |y_2|}$$

$$\therefore (1) \Rightarrow OP = x_1 + \frac{(x_2 - x_1)\,|y_1|}{|y_1| + |y_2|} = \frac{x_1|y_2| + x_2|y_1|}{|y_1| + |y_2|}$$

= Desired value of the root

$= x^{(1)}$, (say).

To find better approximation, we find $y^{(1)}$ from $y^{(1)} = f(x^{(1)})$.

Now, either $y^{(1)}$ and y_1 or y_2 will be of opposite signs. In case $y^{(1)}$ and y_1 are of opposite signs, then one root will lie in the interval $(x_1, x^{(1)})$. We now again apply the method of false position to this interval to get second approximation. In case $y^{(1)}$ and y_2 are of opposite signs, then second approximation is obtained by using the method of false position to the interval $(x^{(1)}, x_2)$.

Continuing this process we can obtain the root to the desired degree of accuracy. //

Note: Regula-falsi method always converse.