Problem sol^n by Bisection Method

Ex. Find the approximate values of the root of the equation

$$3x - \sqrt{(1+\sin x)} = 0$$

using bisection method.

Sol^n The given transcendental eq^n is

$$f(x) = 3x - \sqrt{(1+\sin x)} = 0 \longrightarrow (1).$$

Here, $f(0) = -1$ and $f(1)$

$$\begin{aligned} f(1) &= 3\times 1 - \sqrt{(1+\sin 1^c)} \\ &= 3 - \sqrt{\left(1+\sin \tfrac{180^\circ}{\pi}\right)} \\ &= 3 - \sqrt{(1+0.84147)} \\ &= 3 - \sqrt{(1.84147)} \\ &= 3 - 1.3570 \\ &= 1.6430. \end{aligned}$$

→ by using scientific calculator or by table

$\pi^c = 180^\circ$

$\therefore 1^c = \left(\frac{180}{\pi}\right)^\circ$

Thus, $f(0)$ and $f(1)$ are of opposite signs; therefore at least one real root of $f(x) = 0$ lies in the interval $x = 0$ and $x = 1$.

$\therefore$ First approximation of the root is

$$x_0 = \frac{0+1}{2} = 0.5$$

Again,

$$\begin{aligned} f(0.5) &= 3\times(0.5) - \sqrt{(1+\sin(0.5))} \\ &= 1.5 - \sqrt{\left(1+\sin\left(\tfrac{180}{\pi}\times 0.5\right)\right)} \\ &= 1.5 - \sqrt{(1+0.4794)} = 0.2837 > 0 \end{aligned}$$

$\therefore f(0)$ & $f(0.5)$ are of opposite signs. & so, a root lies in $(0, 0.5)$. So 2^nd approx. of the root is

$$x_1 = \frac{0+0.5}{2} = 0.25$$

Again,

$$f(0.25) = 3\times(0.25) - \sqrt{(1+\sin(0.25))}$$
$$= 0.75 - \sqrt{\left(1 + \sin\left(\frac{180^\circ}{\pi}\times 0.25\right)\right)}$$
$$= 0.75 - \sqrt{(1 + 0.2474)}$$
$$= 0.75 - 1.11687$$
$$= -0.36687 < 0$$

Thus, $f(0.25)$ and $f(0.5)$ have opposite signs.
Hence, at least a root lies in the interval $(0.25, 0.5)$.

$\therefore$ 3rd approx. of a root is

$$x_2 = \frac{0.25 + 0.5}{2} = 0.375$$

Again,

$$f(0.375) = 3\times(0.375) - \sqrt{(1 + \sin(0.375^c))}$$
$$= 1.125 - \sqrt{\left(1 + \sin\left(\frac{180^\circ}{\pi}\times 0.375\right)\right)}$$
$$= 1.125 - \sqrt{(1 + 0.36627)}$$
$$= 1.125 - 1.1688766$$
$$= -0.0438766 < 0$$

Thus $f(0.375)$ & $f(0.5)$ have opposite signs.
Hence, at least a real root lies in the interval $(0.375, 0.5)$. Hence,

4th approx. of the root is

$$x_3 = \frac{0.375 + 0.5}{2} = 0.4375$$

Again, $f(0.4375) = 3\times(0.4375) - \sqrt{(1+\sin 0.4375)} = 0.1193 > 0$

$\therefore f(0.375)\times f(0.4375) < 0 \Rightarrow$ root lies in $(0.375, 0.4375)$.

$\therefore$ 5th approx. is $x_4 = \frac{0.375 + 0.4375}{2} = 0.40625.$