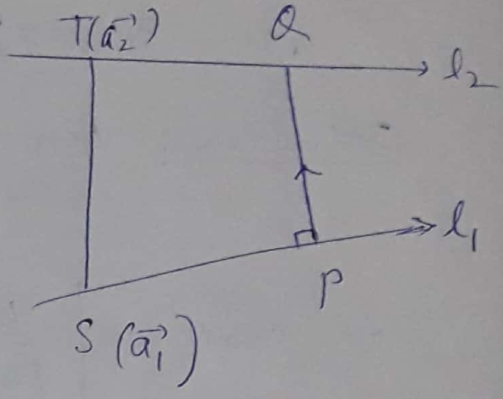
দুটাল বিষম রেখার আন্তর দূরত্ব
(Distance between two skew lines)

Let l_1 and l_2 be two skew lines whose eqns. are

$$\vec{r} = \vec{a_1} + \lambda \vec{b_1} \longrightarrow (1)$$

&

$$\vec{r} = \vec{a_2} + \mu \vec{b} \longrightarrow (2)$$



Then we can show that the shortest distance betn. the two lines

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

Cartesian form: Let the eqns. of the line be

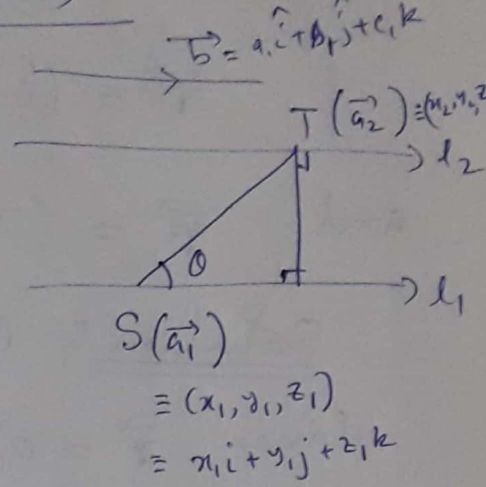
$$l_1 : \frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

$$l_2 : \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$$

Then the shortest distance betn. the lines will be

$$d = \left| \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2 + (a_1 b_2 - a_2 b_1)^2}} \right|$$

সমান্তরাল রেখার মধ্যে দূরত্ব :
(Distance between parallel lines)



Let l_1 and l_2 be two parallel (also co-planner) lines whose eqns. are:

$$\vec{r} = \vec{a_1} + \lambda \vec{b} \longrightarrow (1)$$

and $$\vec{r} = \vec{a_2} + \mu \vec{b} \longrightarrow (2)$$

Then it can be shown that the dist. betn. the lines is

$$d = \left| \frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} \right|$$

Cartesian form: Let the eqns. of the lines be

$$l_1 : \frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

$$l_2 : \frac{x - x_2}{a_1} = \frac{y - y_2}{b_1} = \frac{z - z_2}{c_1}$$

Then the sortest distance between the lines will be

$$d = \left| \frac{\begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \end{vmatrix}}{\sqrt{a_1^2 + b_1^2 + c_1^2}} \right|$$

Exercise 11.2

(17). Find the shortest distance between the lines

$$\vec{r} = (\hat{i}+2\hat{j}+\hat{k}) + \lambda(\hat{i}-\hat{j}+\hat{k})$$

and $$\vec{r} = (2\hat{i}-\hat{j}-\hat{k}) + \mu(2\hat{i}+\hat{j}+2\hat{k}).$$

Solution: Comparing the given lines with the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ respectively, we get

$$\vec{a_1} = i+2j+k, \quad \vec{b_1} = i-j+k$$

$$\vec{a_2} = 2i-j-k, \quad \vec{b_2} = 2i+j+2k$$

$$\therefore \vec{a_2} - \vec{a_1} = (2i-j-k) - (i+2j+k) = i-3j-2k$$

& $$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = i(-2-1) - j(2-2) + k(1+2) = -3i+3k$$

$$\therefore |\vec{b_1}\times\vec{b_2}| = \sqrt{(-3)^2+0^2+3^2} = \sqrt{18} = 3\sqrt{2}$$

$\therefore$ required distance

$$d = \left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right| = \left|\frac{(-3i+3k)\cdot(i-3j-2k)}{3\sqrt{2}}\right|$$

$$= \left|\frac{-3+0-6}{3\sqrt{2}}\right| = \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

Exercise 11.2

(15) Find the shortest distance between the lines:

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \quad \& \quad \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}.$$

Solution: Here,

$$\begin{cases} x_1 = -1, \; y_1 = -1, \; z_1 = -1 \\ x_2 = 3, \; y_2 = 5, \; z_2 = 7 \end{cases}$$

$$\begin{cases} a_1 = 7, \; b_1 = -6, \; c_1 = 1 \\ a_2 = 1, \; b_2 = -2, \; c_2 = 1. \end{cases}$$

$$\begin{matrix} a_1 & b_1 & c_1 & a_1 \\ a_2 & b_2 & c_2 & a_2 \end{matrix}$$

$$\begin{matrix} 7 & -6 & 1 & 7 \\ 1 & -2 & 1 & 1 \end{matrix}$$

Now, $\sqrt{\Sigma (a_1 b_2 - a_2 b_1)^2}$

$$= \sqrt{(-14+6)^2 + (-6+2)^2 + (1-7)^2}$$

$$= \sqrt{8^2 + 4^2 + 6^2}$$

$$= \sqrt{64+16+36} = \sqrt{116}$$

Again, $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

$$= \begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 4(-6+2) - 6(7-1) + 8(-14+6)$$

$$= 4(-4) - 6(6) + 8(-8)$$

$$= -16 - 36 - 64 = -116$$

$$\therefore d = \left| \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\Sigma (a_1 b_2 - a_2 b_1)^2}} \right| = \left| \frac{-116}{\sqrt{116}} \right| = \sqrt{116} = 2\sqrt{29}$$

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