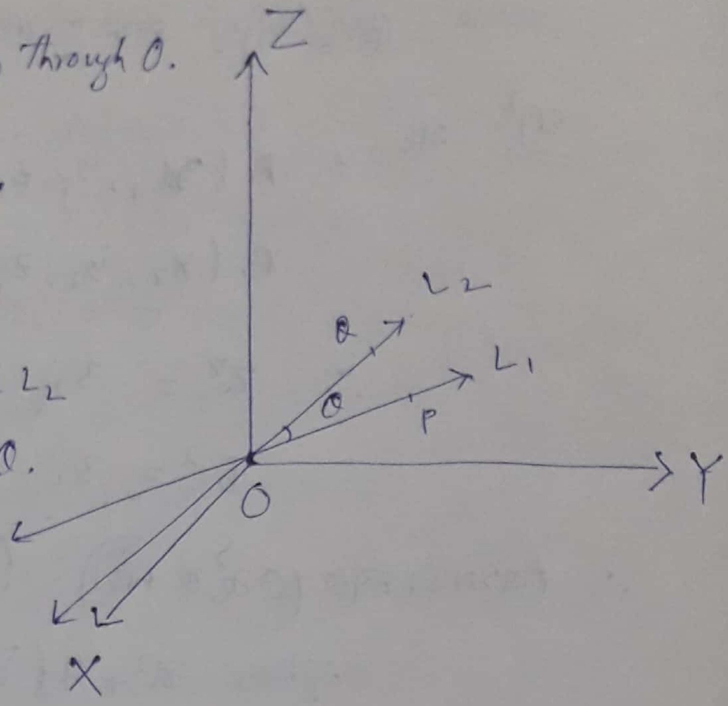
দুডাল ৰেখাৰ মাজৰ কোণ
(Angle between two lines)

Let, L_1 & L_2 be two lines passing through O.

Let, d.r. of $L_1 \longrightarrow a_1, b_1, c_1$.

& d.r. of $L_2 \longrightarrow a_2\ b_2\ c_2$.

Let angle between L_1 and L_2 (i.e. between OP and OQ) be θ.



$\vec{OP}$ ৰ d.r. : a_1, b_1, c_1

$\vec{OQ}$ ৰ d.r. : a_2, b_2, c_2

$\therefore \vec{OP} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{OQ} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$.

Also,

$$\cos\theta = \left| \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \right|$$

$$= \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \right| \longrightarrow (1)$$

Again, in terms of $\sin\theta$:

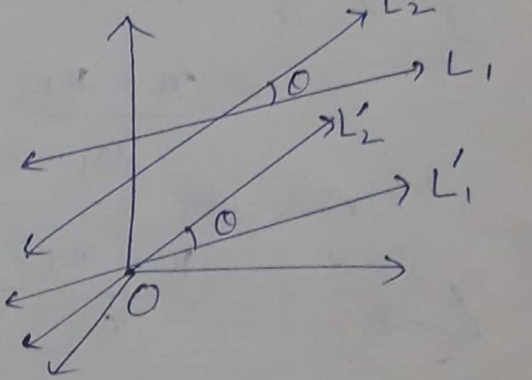
$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1 - \frac{(a_1a_2+b_1b_2+c_1c_2)^2}{(a_1^2+b_1^2+c_1^2)(a_2^2+b_2^2+c_2^2)}}$$

$$= \frac{\sqrt{(a_1^2+b_1^2+c_1^2)(a_2^2+b_2^2+c_2^2) - (a_1a_2+b_1b_2+c_1c_2)^2}}{\sqrt{(a_1^2+b_1^2+c_1^2)(a_2^2+b_2^2+c_2^2)}}$$

or, $$\sin\theta = \frac{\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2}}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{\sqrt{\Sigma(a_1b_2-a_2b_1)^2}}{\sqrt{\Sigma a_1^2}\,\sqrt{\Sigma a_2^2}} \longrightarrow (2)$$

Note (1) If L_1 & L_2 do not passes through origin (0), then we can consider two parallel lines L_1' and L_2' (of L_1 and L_2 respectively) passing through origin O.



Note (2): If

d.c. of $L_1 \rightarrow l_1, m_1, n_1$

& d.c. of $L_2 \rightarrow l_2, m_2, n_2$,

Then $\boxed{\cos\theta = l_1l_2 + m_1m_2 + n_1n_2}$, $\because l_1^2+m_1^2+n_1^2 = 1 = l_2^2+m_2^2+n_2^2$ $\longrightarrow (3)$

& $\sin\theta = \sqrt{\Sigma(l_1m_2-l_2m_1)^2} \longrightarrow (4)$

Also, $\boxed{\theta = 0 \iff a_1a_2+b_1b_2+c_1c_2 = 0}$ & $\boxed{\theta = 0^\circ \iff \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$

Angle between two lines when their eqns. are given:

Let the eqns of the line be

$$\vec{r} = \vec{a_1} + \lambda \vec{b_1} \longrightarrow (1)$$

& $$r = \vec{a_2} + \mu \vec{b_2} \longrightarrow (2)$$

If angle between (1) & (2) be θ, then

$$\cos\theta = \left| \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}||\vec{b_2}|} \right| \longrightarrow (3)$$

Again, if in cartesian form of the lines are:

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \longrightarrow (4)$$

& $$\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2} \longrightarrow (5)$$

Also, if θ be the angles betn them, where a_1, b_1, c_1 & a_2, b_2, c_2 are d.r. of the line (4) & (5) respectively, then

$$\cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \right| \longrightarrow (6)$$

Exercise 11.2

10. (i) Find the angle between the lines

$$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$$

& $$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

Solution: Here, $\vec{b_1} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

& $\vec{b_2} = \hat{i} + 2\hat{j} + 2\hat{k}$

$$\therefore \cos\theta = \left|\frac{\vec{b_1}\cdot\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}\right| = \left|\frac{(3i+2j+6k)\cdot(i+2j+2k)}{\sqrt{9+4+36}\ \sqrt{1+4+4}}\right|$$

$$= \left|\frac{3+4+12}{7\times 3}\right| = \frac{19}{21}$$

$$\therefore \theta = \cos^{-1}\left(\frac{19}{21}\right) \leftarrow \text{Ans.}$$

11. (i) Find the angle between the lines:

$$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \quad \text{and} \quad \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}.$$

Solution: Here, d.r. of 1st line are: 2, 5, −3.

& d.r. of 2nd line are: −1, 8, 4.

$$\therefore \cos\theta = \left|\frac{2\cdot(-1)+5\cdot(8)+(-3)\cdot 4}{\sqrt{2^2+5^2+(-3)^2}\ \sqrt{(-1)^2+8^2+4^2}}\right| = \frac{-2+40-12}{\sqrt{38}\ \sqrt{81}}$$

$$= \frac{26}{9\sqrt{38}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right) \leftarrow \text{Ans.}$$