UNIT-1

ALGORITHMS: Def^n:

An ALGORITHM is a precisely defined sequence of steps for performing a specific task.

CONVERGENCE:

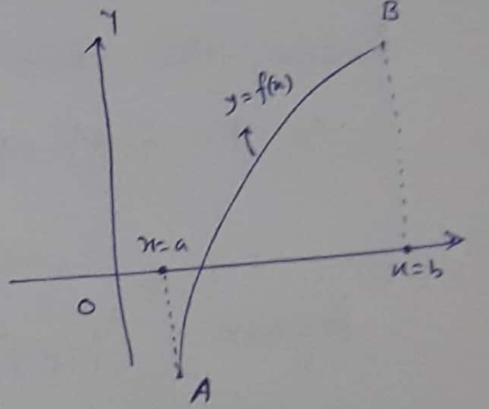
There are many algorithms which are iterative in nature. These algorithms generate a sequence of approximations that converge toward the desired solution. When several techniques are available for solving a particular problem, we generally choose a technique whose sequence converges as rapidly as possible.

Solution of Polynomial and Transcendental Equations:

There are various methods of solving such equations. With the help of these methods we first find a first approximation to a root of the given equation and then successively improve it.

(1) Bisection Method (Bolzano Method)

If a function $f(x)$ is continuous between 'a' and 'b' and $f(a)$ and $f(b)$ are opposite signs, then there exists at least one root between 'a' and 'b'.



Let $f(a)$ be negative and $f(b)$ be positive so that the approximate value of the root between them is $x_0 = \frac{a+b}{2}$. If $f(x_0) = 0$, then it asserts that x_0 is the correct root of $f(x) = 0$. On the other hand if $f(x_0) \neq 0$, then the root either lies in $\left(a, \frac{a+b}{2}\right)$ or $\left(\frac{a+b}{2}, b\right)$ depending on whether $f(x_0)$ is positive or negative. We again bisect the interval and repeat the process until the root is obtained to desired accuracy.

Ex. Find a real root of the equation

$$f(x) = x^3 - x - 1 = 0$$

by bisection method.

Sol^n Given eq^n. is

$$f(x) = x^3 - x - 1 = 0.$$

Here,

$$f(1) = 1 - 1 - 1 = -1 < 0$$

$$f(2) = 8 - 2 - 1 = 5. > 0$$

∵ $f(1)$ & $f(2)$ have opposite ~~root~~ signs, hence the root lies between $(1, 2)$

i.e. the first approximation is

$$x_0 = \frac{1+2}{2} = 1.5.$$

Again, $f(1.5) = (1.5)^3 - 1.5 - 1 = 0.875 > 0$.

Thus $f(1)$ & $f(1.5)$ have opposite signs.

∴ roots lies between 1 and 1.5.

∴ 2nd approximation is

$$x_1 = \frac{1+1.5}{2} = 1.25$$

Again, $f(1.25) = (1.25)^3 - 1.25 - 1 = -0.546875 < 0$

Since $f(1.25)$ & $f(1.5)$ have opposite signs, so the roots lies bet^n 1.25 & 1.5.

∴ 3rd approx. is $x_2 = \dfrac{1.25 + 1.5}{2} = 1.375$

Now,

$$f(1.375) = (1.375)^3 - 1.375 - 1 = 0.224609 > 0$$

$\because$ $f(1.25)$ & $f(1.375)$ have opposite signs, hence root lies in $(1.25, 1.375)$.

Thus 4^{th} approximation is

$$x_3 = \frac{1.25 + 1.375}{2} = 1.3125$$

Again,

$$f(1.3125) = (1.3125)^3 - 1.3125 - 1 = -0.5151367 < 0$$

Since $f(1.3125)$ & $f(1.375)$ have opposite sings, so the root lies betn $(1.3125, 1.375)$.

Thus, 5^{th} approximation of the root is

$$x_4 = \frac{1.3125 + 1.375}{2} = 1.34375$$

Again, $f(1.34375) = (1.34375)^3 - 1.34375 - 1$
$= 0.0826 > 0$

Hence, root lies in $(1.3125, 1.34375)$.

So, 6^{th} approximation of the root is

$$x_5 = \frac{1.3125 + 1.34375}{2} = 1.328125$$

which is the approximate value of the root. #

Note: This process can be continued as long as required for more & more accurate root.

H.W. Ex. Solve $x^3 - 9x + 1 = 0$ for the root betn $x = 2$ and $x = 4$ by bisection method.