29.10.20

88, Ex. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rule and $\frac{3}{8}$ rule. Hence obtain approximate value of π each case

[(n → even) [n = 3×m, m = 1, 2, 3, ... → no. of sub-division]]

Solution:- Taking $n = 6$, so that

$$nh = 1 - 0 = 1$$

$$\therefore h = \frac{1}{6}.$$

$$\therefore x = 0, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, 1.$$

Now we form the following table

x	$f(x)$
0	$\frac{1}{1+0} = 1 = 1.00000 = y_0$
$\frac{1}{6}$	$\frac{1}{1+(\frac{1}{6})^2} = \frac{36}{37} = 0.97297$
$\frac{2}{6}$	$\frac{1}{1+(\frac{2}{6})^2} = \frac{36}{40} = 0.90000$
$\frac{3}{6}$	$\frac{1}{1+(\frac{3}{6})^2} = \frac{36}{45} = 0.80000$
$\frac{4}{6}$	$\frac{1}{1+(\frac{4}{6})^2} = \frac{36}{52} = 0.69231$
$\frac{5}{6}$	$\frac{1}{1+(\frac{5}{6})^2} = \frac{36}{61} = 0.59016$
1	$\frac{1}{1+1^2} = \frac{1}{2} = 0.50000$

Now, by Simpson's $\frac{1}{3}$rd rule

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{1}{18}\left[(1.00000+0.50000)+4(0.97297+0.80000+0.59016)+2(0.90000+0.69231)\right]$$

$= 0.785397 \longleftarrow$ Ans.

Again by Simpson's $\frac{3}{8}$th rule.

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3h}{8}\left[(y_0+y_6)+2(y_3)+3(y_1+y_2+y_4+y_5)\right]$$

$$= \frac{1}{16}\left[(1.0000+0.50000)+2\times 0.80000 + 3(0.97297+0.90000+0.69231+0.59016)\right]$$

$= 0.785395 \longleftarrow$ Ans.

H.W. Evaluate: $\int_1^3 \frac{dx}{x}$ by taking 8 sub-~~division~~ intervals (i.e. $\frac{1}{3}$rd rule).

Sol^n.

Now by Simpson's $\frac{1}{3}$rd rule, we have,

$$\int_0^1 \frac{1}{1+x^2}\,dx = 0.785397$$

$$\Rightarrow \left[\tan^{-1} x\right]_0^1 = 0.785347$$

$$\Rightarrow \pi/4 = 0.785347$$

$$\Rightarrow \pi = 3.141588$$

Also, we have by Simpson's $\frac{3}{8}$th rule

$$\frac{\pi}{4} = 0.785395$$

$$\Rightarrow \pi = 3.141580.$$

H.W. Ex. Evaluate $\int_1^3 \frac{dx}{x}$ by taking 8 sub-intervals (i.e., $\frac{1}{3}$rd rule).

Solⁿ: Taking $n = 8$ so that

$$nh = 3-1 = 2$$

$$\therefore h = \frac{2}{8} = \frac{1}{4}.$$

$$\therefore x = 1, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}, \frac{9}{4}, \frac{10}{4}, \frac{11}{4}, 3.$$

Now, we form the following table

x	$y = f(x)$
$x_0 = 1$	$y_0 = \frac{1}{1} = 1 = 1.00000$
$x_1 = 5/4$	$y_1 = \frac{1}{5/4} = 4/5 = 0.80000$
$x_2 = 6/4$	$y_2 = 4/6 = 0.66666$
$x_3 = 7/4$	$y_3 = 4/7 = 0.57148$
$x_4 = 8/4$	$y_4 = 4/8 = 0.50000$
$x_5 = 9/4$	$y_5 = 4/9 = 0.44444$
$x_6 = 10/4$	$y_6 = 4/10 = 0.40000$
$x_7 = 11/4$	$y_7 = 4/11 = 0.36364$
$x_8 = 3$	$y_8 = 3 = 0.3333$

Now, by Simpson's $\frac{1}{3}$rd rule, we have

$$\int_1^3 \frac{dx}{x} = \frac{1}{3}h\left[(y_0+y_8)+4(y_1+y_3+y_5+y_7) + 2(y_2+y_4+y_6)\right]$$

$$= \frac{1}{3}\cdot\frac{1}{4}\left[(1+0.33333)+4(0.80000+0.57143+0.44444+0.36364) + 2(0.66666+0.5+0.4)\right]$$

$$= \frac{1}{12}\left[1.33333+4\times(2.17951) + 2\times(1.56666)\right]$$

$$= \frac{13.184659}{12}$$

$$= 1.09872.$$

Ex. Evaluate

$$I = \int_0^1 \frac{dx}{1+x}$$

given

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
$\frac{1}{x}$	1.0 y_0	.90 $=y_1$	.83 $=y_2$	.76 $=y_3$	.71 $=y_4$	.66 $=y_5$	.62 $=y_6$	.58 y_7	.55 y_8	.52 y_9	.5 y_{10}

put

$1+x = u$ when $x=0,\ u=1$,

$dx = du$,, $x=1,\ u=2$

$$\therefore I = \int_0^1 \frac{dx}{1+x} = \int_1^2 \frac{du}{u} = \int_1^2 \frac{dx}{x}$$

Here $h = .1$ & $y = f(x) = \frac{1}{x}$.

Now, By Simpson's $\frac{1}{3}$rd rule,

$$\int_1^2 \frac{dx}{x} = \frac{.1}{3}\left[(y_0+y_{10}) + 4(y_1+y_3+y_5+y_7+y_9) + 2(y_2+y_4+y_6+y_8)\right]$$

$$= \frac{.1}{3}\left[(1.0+.5) + 4(.90+.76+.66+.58+.52) + 2(.83+.71+.62+.55)\right]$$

$$= \frac{.1}{3}\left[1.5 + 4(3.42) + 2(2.71)\right]$$

$$= \frac{.1}{3}(1.5 + 13.68 + 5.42)$$

$$= \frac{.1}{3}(20.60)$$

$$= \frac{2.060}{3} = 0.68\dot{6}$$

$= 0.69$ (approx).