— x —

Simpson's $\frac{1}{3}$rd & $\frac{3}{8}$ rule:→

We have, General Quadrature formula :-

$$\int_{x_0}^{x_0+nh} f(x)\,dx = h\left[ny_0 + \frac{n^2}{2}\Delta y_0 + \frac{\frac{n^3}{3}-\frac{n^2}{2}}{2!}\Delta^2 y_0 + \frac{\frac{n^4}{4}-n^3+n^2}{3!}\Delta^3 y_0 + \cdots \text{upto } (n+1) \text{ term}\right.$$

'87 11(a) **Simpson's $\frac{1}{3}$ rule:** Putting $n=2$ in General Quadrature formula and neclecting 3rd and higher differences, we get

$$\int_{x_0}^{x_0+2h} f(x)\,dx = h\left[2y_0 + 2(y_1-y_0) + \frac{\frac{8}{3}-2}{2}(y_2-2y_1+\right.$$

$$= h\left[2y_1 + \frac{1}{3}(y_2-2y_1+y_0)\right]$$

$$= \frac{h}{3}\left[6y_1 + y_2 - 2y_1 + y_0\right]$$

$$= \frac{h}{3}\left[y_0 + 4y_1 + y_2\right].$$

Similarly,

$$\int_{x_0+2h}^{x_0+4h} f(x)\,dx = \frac{h}{3}\left[y_2 + 4y_3 + y_4\right]$$

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Lastly,

$$\int_{x_0+(n-2)h}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[y_{n-2} + 4y_{n-1} + y_n\right].$$

Adding all the integrals,

$$\int_{x_0}^{x_0+2h} f(x)\,dx + \int_{x_0+2h}^{x_0+4h} f(x)\,dx + \cdots + \int_{x_0+(n-2)h}^{x_0+nh} f(x)\,dx$$

$$= \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots+y_{n-1}) + 2(y_2+y_4+\cdots+y_{n-2})\right]$$

i.e. $\Rightarrow$ $$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots+y_{n-1}) + 2(y_2+y_4+\cdots+y_{n-2})\right]$$

<u>Simpson's $\frac{3}{8}$ rule</u> :- Putting $n=3$ in General Quadrature formula & neglecting 4-th and higher differences, we get-

$$\int_{x_0}^{x_0+3h} f(x)\,dx = h\left[3y_0 + \frac{9}{2}(y_1-y_0) + \frac{9-\frac{9}{2}}{2}(y_2-2y_1+y_0) + \frac{\frac{81}{4}-27+9}{6}(y_3-3y_2+3y_1-y_0)\right]$$

$$= h\left[3y_0 + \frac{9}{2}y_1 - \frac{9}{2}y_0 + \frac{9}{4}(y_2) - \frac{18}{4}y_1 + \frac{9}{4}y_0 + \frac{3}{8}y_3 - \frac{9}{8}y_2 + \frac{9}{8}y_1 - \frac{3}{8}y_0\right]$$

$$= h\left[\frac{3}{8}y_0 + \frac{9}{8}y_1 + \frac{9}{8}y_2 + \frac{3}{8}y_3\right]$$

$$= \frac{3}{8}h\left[y_0 + 3y_1 + 3y_2 + y_3\right]$$

Similarly, $$\int_{x_0+3h}^{x_0+6h} f(x)\,dx = \frac{3}{8}h\left[y_3+3y_4+3y_5+y_6\right]$$

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Lastly, $$\int_{x_0+(n-3)h}^{x_0+nh} f(x)\,dx = \frac{3}{8}h\left[y_{n-3}+3y_{n-2}+3y_{n-1}+y_n\right]$$

Adding all the Integrals, we get-

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+y_7+y_8+y_{10}+\cdots \cdots + y_{n-1})+2(y_3+y_6+y_9+\cdots+y_{n-3})\right]$$

x

Ex. Given $e^0=1$, $e^1=2.72$, $e^2=7.39$, $e^3=20.09$, $e^4=54.60$.

Evaluate $\int_0^4 e^x\,dx$, by simpson's 1/3 rule.

Also find the error.

[Note: use 1/3 rule when n = 2, 4, 6, ...

" 3/8 " " n = 3, 6, 9, ...]

Sol^n: Here $a=0$, $b=4$

$\therefore nh = b-a = 4-0 = 4$

& $n = 4$. (no. of division of interval).

$\therefore h = 1$.

$b = x_0 + nh$

$= a + nh$.

By Simpson's 1/3 rule

$$\int_0^4 e^x\,dx = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2(y_2)\right]$$

$$= \frac{1}{3}\left[1+54.60+4(2.72+20.09)+2\times 7.39\right]$$

$= 53.873$.

Again actual value is

$$\int_0^4 e^x\,dx = e^4 - e^0$$

$= 54.60 - 1$

$= 53.60$

$\therefore$ error = Actual value − obtain value

$= 53.60 - 53.873$

$= -0.273$. //