

Numerical differentiation. [~~scribble~~].

Ex(1) Find the first two derivatives of $f(x)$ at $x=2$
if $f(0) = 1, f(1) = 0, f(2) = 1, f(3) = 10$

{ [Hint:- Here intervals are equal, \therefore apply Newton's forward formula]
(when intervals are unequal, apply Newton's divided diff. formula) }

Solⁿ We form the difference table as

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	1			
1	0	-1	2	
2	1	1	8	6
3	10	9		

Now, Newton's forward formula is

$$f(x) = f(0) + x \cdot \Delta f(0) + \frac{x(x-1)}{2} \Delta^2 f(0) + \frac{x(x-1)(x-2)}{6} \Delta^3 f(0) \quad \left\{ \begin{array}{l} u = \frac{x-a}{h} \\ \therefore u = \frac{x-0}{1} \\ \therefore u = x \end{array} \right.$$

$$= 1 + x \cdot (-1) + \frac{x(x-1)}{2} \cdot 2 + \frac{x(x-1)(x-2)}{6} \cdot 6$$

$$= 1 - x + x^2 - x + x(x^2 - 3x + 2)$$

$$= 1 - 2x + x^2 + x^3 - 3x^2 + 2x$$

$$= x^3 - 2x^2 + 1$$

$$\therefore f'(x) = 3x^2 - 4x$$

$$f''(x) = 6x - 4$$

$$\therefore f'(2) = 3 \times 4 - 4 \times 2 = 4$$

$$f''(2) = 6 \times 2 - 4 = 8$$

Ex. (ii) Find the first two derivatives of $f(x)$ at $x=2$, if

$$f(0) = 2, f(1) = 3, f(2) = 12, f(5) = 147.$$

[Hint: Unequal interval, \therefore apply N.D.D. formula].

Sol.ⁿ Let us form the following divided difference table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	2			
1	3	$\frac{3-2}{1-0} = 1$		
2	12	$\frac{12-3}{2-1} = 9$	$\frac{9-1}{2-0} = 4$	
5	147	$\frac{147-12}{5-2} = 45$	$\frac{45-9}{5-1} = 9$	$\frac{9-4}{5-0} = 1$

Now, by Newton's divided difference formula

$$f(x) = f(x_0) + (x-x_0)f(x_0, x_1) + (x-x_0)(x-x_1)f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0, x_1, x_2, x_3)$$

$$= 2 + (x-0) \cdot 1 + (x-0)(x-1) \cdot 4 + (x-0)(x-1)(x-2) \cdot 1$$

$$= 2 + x + 4x^2 - 4x + x^3 - 3x^2 + 2x$$

$$= x^3 + x^2 - x + 2$$

$$\therefore f'(x) = 3x^2 + 2x - 1, f''(x) = 6x + 2$$

$$\therefore f'(2) = 3 \times 4 + 2 \times 2 - 1 = 15$$

$$f''(2) = 6 \times 2 + 2 = 14$$

} ← Ans