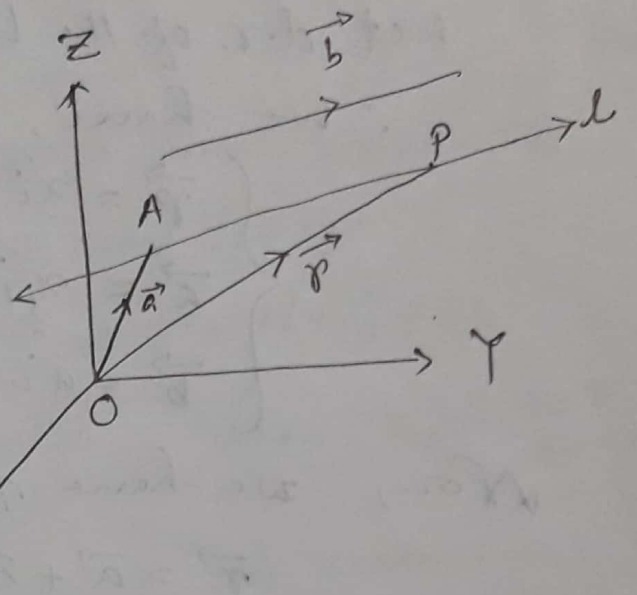


অনুঘাতক ও ডাল রেখার সমীকরণ :
(Equation of a line in Space)

① এক নির্দিষ্ট-বিন্দুতে (যা বা) এবং এক নির্দিষ্ট-দিকের (যেই \vec{b}) সমান্তরাল ও ডাল রেখার সমীকরণ : (Equation of a line through a given point and parallel to a given vector \vec{b})

সমা: l রেখাডাল \vec{b} এর সমান্তরাল
 l রেখা $A(\vec{a})$ বিন্দুতে যাক।
 যেকোন $P(\vec{r})$ বিন্দু l -তে আছে
 বিবেচনা করি।



[Let the line l pass through $A(\vec{a})$ and parallel to \vec{b} . Let $P(\vec{r})$ be any point on l .]

Now, $\vec{AP} \parallel \vec{b}$

$\therefore \vec{AP} = \lambda \vec{b} \rightarrow (1)$

$\Rightarrow \vec{r} - \vec{a} = \lambda \vec{b}$

$\Rightarrow \boxed{\vec{r} = \vec{a} + \lambda \vec{b}} \rightarrow (2)$

which is the required eqn: #

Remark:

If $\vec{b} = ai + bj + ck$, then a, b, c are d.r. of the line.

Conversely, (বিপরীত অর্থে), If a, b, c are d.r. of a line, then the vector $\vec{b} = ai + bj + ck$ is parallel to the line.

ଉତ୍ପତ୍ତି ଅନୁସାରେ ପାରା ମାଟ୍ରିକାଲ୍ ରୂପ ନିର୍ଣ୍ଣୟ :
(Derivation of cartesian form from vector form)

Let $A \equiv A(x_1, y_1, z_1)$
 $P \equiv P(x, y, z)$.

Let d.r. of the line are: a, b, c .

∴ We have,

$$\begin{cases} \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \\ \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \\ \vec{b} = a\hat{i} + b\hat{j} + c\hat{k} \end{cases}$$

Now, we have from (2),

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} + \lambda(a\hat{i} + b\hat{j} + c\hat{k}) \\ = (a_1 + \lambda a)\hat{i} + (a_2 + \lambda b)\hat{j} + (a_3 + \lambda c)\hat{k}$$

Comparing the co-efficients of i, j and k we have

$$\begin{cases} x = a_1 + \lambda a \\ y = a_2 + \lambda b \\ z = a_3 + \lambda c \end{cases}$$

$$\Rightarrow \begin{cases} \frac{x - a_1}{a} = \lambda \\ \frac{y - a_2}{b} = \lambda \\ \frac{z - a_3}{c} = \lambda \end{cases}$$

Hence, we get (eliminating λ): $\boxed{\frac{x - a_1}{a} = \frac{y - a_2}{b} = \frac{z - a_3}{c}}$

which is the required eqn. in parametric form.
(ପାରାମିଟ୍ରିକ୍ ରୂପ)

Note: ସେମାନଙ୍କ ଦିଗାଙ୍କ (d.c.) l, m, n ର ସମତ-ସମ୍ପର୍କ:

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

Exercise 11.2

Q. No. (4) $(1, 2, 3)$ ବିନ୍ଦୁ-ଆକାର (ସ୍ଥାନ) ଗଠ 3i+2j-2k
ଦିଗ-ସମ୍ପର୍କ ସମ୍ପର୍କରେ ସମୀକରଣ ଉଲିଙ୍ଗନ

Sol: Here, $(x_1, y_1, z_1) = (1, 2, 3) \Rightarrow \vec{a} = 1\hat{i} + 2\hat{j} + 3\hat{k}$
 $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$, (here, $a=3, b=2, c=-2$)

∴ eqn. of line:

$$\vec{r} = \vec{a} + \lambda \vec{b}, \quad \lambda \text{ is scalar.}$$

$$\Rightarrow \vec{r} = (i+2j+3k) + \lambda(3i+2j-2k) \leftarrow \text{Ans.}$$

which is vector eqn. of the line.

Note: କାର୍ଡିନାଟ୍ ସମୀକରଣ (Cartesian eqn.):

We know, cartesian eqn is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$$\Rightarrow \frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2} \leftarrow \text{Ans.}$$

