Central Difference Interpolation formula:

Introduction: Newton's General Interpolation formula as well as Newton's forward & ~~fo~~ backward formula are applicable to nearly all cases of interpolation. But they do not converge as rapidly as the central difference formula. These formulae are based on differences obtained from the values of $f(x)$ on either side of the origin.

Central difference operator:

The central difference operator (δ) is defined as

$$\boxed{\delta \equiv E^{1/2} - E^{-1/2}}$$

∴ the first central difference

$$\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$$

$$= \left(E^{1/2} - E^{-\frac{1}{2}}\right) f(x), \quad \text{where } E = 1+\Delta$$

$$= E^{-1/2}(E-1) f(x)$$

$$= E^{-\frac{1}{2}} \Delta f(x)$$

$$\Rightarrow \boxed{\delta E^{1/2} f(x) = \Delta f(x)} \longrightarrow (*)$$

Thus,

$$\delta \equiv E^{\frac{1}{2}} - E^{-\frac{1}{2}} = E^{-\frac{1}{2}}\Delta \longrightarrow (1)$$

Again, we know,

$$\nabla f(x) = f(x) - f(x-h) = (1-E^{-1}) f(x).$$

i.e. $\nabla \equiv (1-E^{-1})$

$$\Rightarrow \nabla E^{1/2} = (1-E^{-1})E^{1/2} = E^{1/2} - E^{-1/2} = \delta \longrightarrow (2)$$

∴ (1) & (2) $\Rightarrow$ $\boxed{\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{1/2}} \longrightarrow (3)$

The n^{th} power of these operator applied to y_x gives

$$\delta^n y_x = \Delta^n E^{-n/2} y_x = \nabla^n E^{n/2} y_x$$

or, $$\delta^n y_x = \Delta^n y_{x-n/2} = \nabla^n y_{x+n/2}.$$

The average operator:

The average operator μ is defined by

$$\boxed{\mu \equiv \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]}$$

Central Difference Table :→

(using ⊛: $\Delta f(x) = \delta E^{1/2} f(x) = \delta f(x+\tfrac{1}{2})$, taking $h=1$)

x	y_x	δy_x	$\delta^2 y_x$	$\delta^3 y_x$	$\delta^4 y_x$
-2	y_{-2}	$\Delta y_{-2} =$			
		$\delta y_{-3/2}$	$\Delta^2 y_{-2} = \delta^2 y_{-1}$	$\Delta^3 y_{-2} =$	
-1	y_{-1}	$\Delta y_{-1} =$		$\delta^3 y_{-1/2}$	$\Delta^4 y_{-2} =$
		$\delta y_{-1/2}$	$\Delta^2 y_{-1} =$		$\delta^4 y_0$
0	y_0	$\Delta y_0 =$	$\delta^2 y_0$	$\Delta^3 y_{-1} =$	
		$\delta y_{1/2}$	$\Delta^2 y_0 =$	$\delta^3 y_{1/2}$	
1	y_1	$\Delta y_1 =$	$\delta^2 y_1$		
		$\delta y_{3/2}$			
2	y_2				

Note: $\Delta f(x) = \delta E^{1/2} f(x)$

$\Rightarrow \Delta y_{-2} = \delta E^{1/2} y_{-2}$

$= \delta y_{-2+1} = \delta y_{-3/2}$

$\Delta f(x) = \delta E^{1/2} f(x)$

$\Rightarrow \Delta(\Delta f(x)) = \Delta(\delta y_{-3/2})$

$= \delta E^{1/2}(\delta y_{-3/2})$

$= \delta^2 E^{1/2} y_{-3/2} = \delta^2 y_{-1}$