Relation between the electrical and Thermal Conductivities of Metals. WIEDMANN - FRANZ LAW

It is found that all good conductors of electricity are also good conductors of heat. From this in 1853 Wiedemann and Franz gave the emperical law which states that "The ratio of the thermal and electrical conductivities at a particular Tempr is the same for all metals." In 1872 Lorentz extended the law and showed that this ratio is proportional to the absolute Tempr.

Drude tried to explain this remarkable remark by ~~free electron~~ considering the free electrons in the metals are responsible for both the thermal and electrical conductivities. He again assumed that these electrons inside the metal behave like gas molecules and ~~because~~ because they collide and are in equilibrium with the atoms of the metal and from equipartition of energy, they will have the same energy per degree of freedom as is possessed by the atom.

Let us assume the conduction of electricity in a metalic wire. If X be the electric intensity along the wire, The force of each electron $= Xe$.
where e = charge of an electron. $\longrightarrow (1)$

$\therefore$ Acceleration of an electron $= \frac{Xe}{m}$ where m = mass of an electron.

If λ be the mean free path and $\bar{c}$ the mean velocity of the electron, the average time between two successive collision $= \lambda / \bar{c}$ ——— (2)

Because the velocity at the beginning of the path is zero hence the velocity at its end $= \frac{Xe}{m} \cdot \frac{\lambda}{\bar{c}}$ $\longrightarrow (3)$

$\longrightarrow$

Thus the average drift velocity of an electron,

$$u = \frac{1}{2}\frac{Xe}{m}\cdot\frac{\lambda}{\bar{c}} \longrightarrow (4)$$

If n be the number of electron per unit volume, the no. of electron crossing any section of the conductor having unit cross-sectional area in unit time $= nu$

Hence the electric current i what is charge crossing any section per unit time is given by

$$i = neu = ne\frac{1}{2}\frac{Xe}{m}\cdot\frac{\lambda}{\bar{c}} \longrightarrow (5)$$

Hence the electrical conductivity σ of unit length and unit cross-sectional area can be written as

$$\sigma = \frac{i}{X} = \frac{1}{2}\frac{ne^2\lambda}{m\bar{c}} \longrightarrow (6)$$

We know that from equipartition of energy

$\frac{1}{2}m\bar{c}^2 = \frac{3}{2}KT$ where K = Boltzmann const.

$$m\bar{c}^2 = 3KT.$$

$$\therefore m\bar{c} = \frac{3KT}{\bar{c}} \longrightarrow (7)$$

using (7) in (6)

$$\sigma = \frac{1}{6}\frac{ne^2\lambda\bar{c}}{KT} \longrightarrow (8)$$

Because the electrons behave like gas molecules, hence the thermal conductivity K for the electron gas is given by

$K_e = \eta C_v$ where η = Coefficient of viscosity

and C_v = specific heal of electron gas.

$$\therefore K_e = \frac{1}{3}mn\bar{c}\lambda C_v \qquad \left[\because \eta = \frac{1}{3}mn\bar{c}\lambda\right]$$

$$\therefore C_v = \frac{1}{m}\frac{dE}{dT}$$

$$\therefore K_e = \frac{1}{3}mn\bar{c}\lambda\frac{1}{m}\frac{dE}{dT} \longrightarrow (9)$$

$$\therefore E = \frac{3}{2}KT.$$

$$\therefore \frac{dE}{dT} = \frac{3}{2}K. \longrightarrow (10)$$

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using (10) in (9)

$$K_e = \frac{1}{3} m n \bar{c} \lambda \frac{1}{m} \frac{3}{2} K'.$$

$$\therefore K_e = \frac{1}{2} n \bar{c} \lambda K \longrightarrow (10)$$

If K_e is expressed in calories

$$K_e = \frac{\frac{1}{2} n \bar{c} \lambda K}{J} \longrightarrow (11)$$

Where J = mechanical equivalent of heat.

Now $\frac{(11)}{(8)}$ we get

$$\frac{K_e}{\sigma} = \frac{\frac{1}{2} n \bar{c} \lambda K}{J} \times \frac{6KT}{n e^2 \lambda \bar{c}}$$

$$\therefore \frac{K_e}{\sigma} = \frac{3}{J} \left(\frac{K}{e}\right)^2 T \longrightarrow (12)$$

The relation shows that $\frac{K}{\sigma}$ is independent of the nature of the metals ~~is po~~ and is proportional to the absolute Tempr. The eqn (12) is called Wiedemann-Franz Relation

(Fermi Energy level ↦ The value of energy upto which all the energy states in a Fermi gas at 0 K and above which all the energy states are empty is known as Fermi Energy ~~level~~. and Corresponding energy level is called Fermi Energy level.