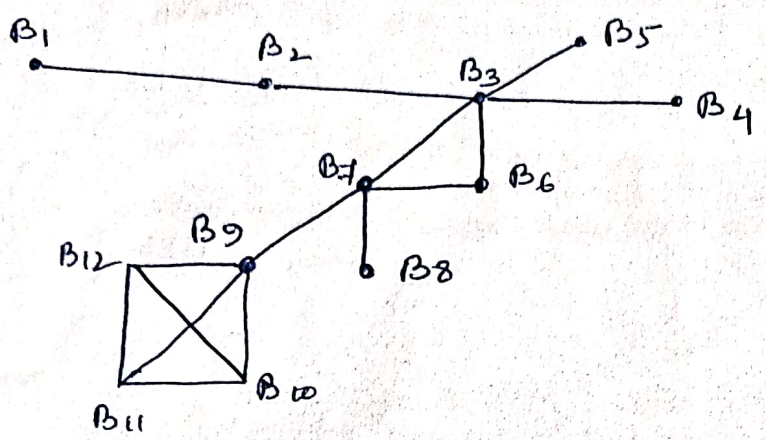


B(G).

Thm: A graph H is the block graph of some graph if and only if every block of H is complete.

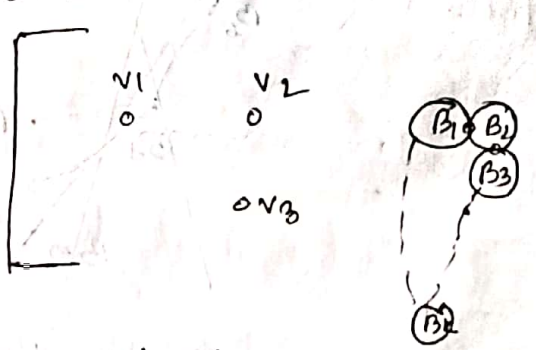
Pf: ~~Copy~~ Copy - P-58

Thm 5: A graph H is a block graph of some graph iff every block of H is complete.

Pf: Let us 1st assume that H be a block graph Then we get a graph G such that H is (the) H=B(G)

Let us assume that there is a block H_1 of H which is not complete. Then there are at least two points in H_1 which are non-adjacent and lie on a common cycle J of length at least 4.

Let $v_1, v_2, \ldots, v_k$, $k \geq 4$ be the points of H_1 which lie on the cycle J. Let $B_1, B_2, \ldots, B_k$ be the corresponding blocks of G which corresponds to $v_1, v_2, \ldots, v_k$. Then $B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_k$ in G is connected and has no cut point (because H_1 is a block corresponds to H_1 & so B has no cut point). Therefore $B_1 \cup B_2 \cup \ldots \cup B_k$ is contained in a block B. Now each $B_i \subset B$ leads to the contradiction to the fact that every B_i is a block (therefore every block

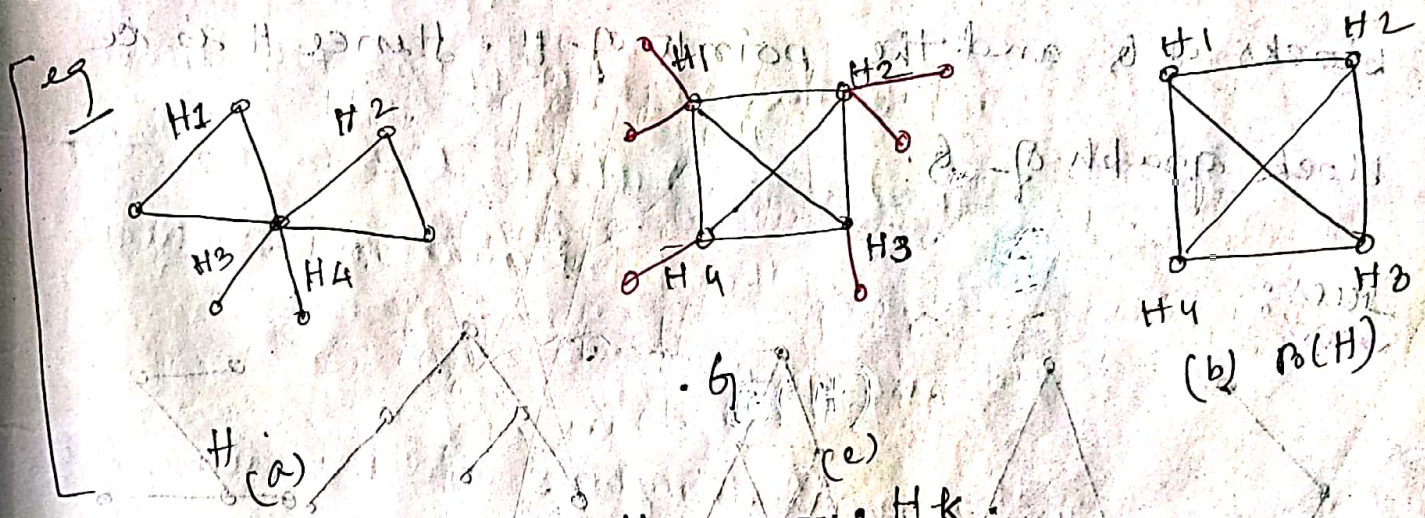


& $B = B_1 \cup B_2 \cup \ldots \cup B_k \cup \ldots$

~~B_i is a subgraph of a block B_j~~) ie this contradicts the maximality of a block. Hence H_i must be complete

Conversely, suppose H be a given graph in which every block is complete. Let $H_1, H_2, \ldots, H_k$ be the blocks of H. Then every point of $B(H)$ corrosponds to some block H_i of H. we may denote the

eg



pts of $B(H)$ by $H_1, H_2, \ldots, H_k$.

Now, to each point H_i of $B(H)$, add end lines equal to the number of points of the block H_i which are not cut point of H. then we get a new graph G from $B(H)$ by adjoing such end lines. we see that every end line is a block of G and corrosponds to a point of H

which is not a cutpoint. Again, if two blocks Hi and Hj are connected in H through a cutpoint v, then the corrosponding points Hi & Hj of $B(H) \subset G$ are adjacent. Then the line Hi & Hj of G is contained in a block B of G. (we see that the block B of G corrosponds to the cutpoint v in H). In this way we get a 1-1 corrospondence bet^n the blocks of G and the points of H. Hence H is a block graph of G.