

## Classical Distribution Law

### ② Microstates and Macrostates :-

An ensemble consisting of a large number of independent systems as a gas consisting of a large number of molecules in phase space. The state of an individual system or molecule may be represented in phase space by a point known as phase point or representative representative point. The phase space may be divided into cells 1, 2, 3, ...  $i$ . A phase ~~too~~ point for any system or molecule may be reside in one of these cells.

"Each arrangement of specified system or molecules with their representative points in particular cells is called a microstate. In other words a microstate of the ensemble may be defined by the specification of the individual position of phase points for each system or molecule of the system". Thus in a microstate we have to state to which cell each system or molecule belongs temporarily.

The gross behaviour the A macrostate defined by  $\Gamma$  each cell is only and  $\Omega$  of molecules.

A  $\Gamma$  in phase space because the point of view it is  $\Omega$  in a particular Thus may cover

Let the cell  $i$  in

Suppose there is  $n_i$  in cell

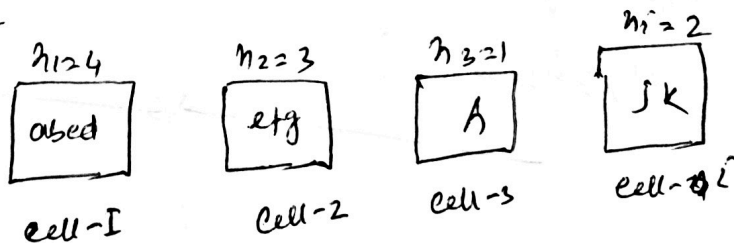
The gross observable properties of macroscopic behaviour that occupying by the specific cells.

A macroscopic macrostate of the ensemble may be defined by the specification of phase point in each cell i.e. by specifying the numbers only and overlooking the identities of the systems of molecules.

A change of phase points between two cells in phase space shifts the position of unit cell because the microstate changes. But macroscopic point of view the exchange makes no difference because it is immaterial which of the molecules are in a particular cell specified by numbers 1, 2, ... etc.

Thus many different microstates which may corresponding to form macrostate.

Let there are cell 1, cell 2, cell 3, ... etc cell  $i$  in phase space



Suppose there four phase points abcd in cell-1 three in phase points in etg in cell-2, one phase point in cell-3, and two phase points JK in cell  $i$ . The macrostate of figure is specified by

merely bring the phase points  $\lambda_1=4, \lambda_2=3$   
- -  $n_i=2$  of different cells. This also constitute  
individual microstate in cell-1, cell-2 and  
so on.

Now if the two phase points  $a$  &  $e$  form  
different cells are interchanged, then the microstate  
is changed because the position of two phase  
points are changed, while the macrostate remain  
constant. Similarly, with the same macrostate  
we can consider different microstate.

The microstate which are allowed  
under given restriction are called accessible  
microstate. One of the most fundamental  
postulate of st. mechanics is that all  
accessible microstate corresponding  
to possible microstate are equally  
probable.

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## \* STIRLING'S

The calculation  
for large value  
gives the approx  
 $n$  is given by

$$\log n!$$

By the defn

$$n! =$$

$$\therefore \log n$$

Now we plot  
ordinates to  
then approximation  
replacement by  
range where  
slightly we  
this approx

## \* STIRLING'S APPROXIMATION $\frac{0}{0}$

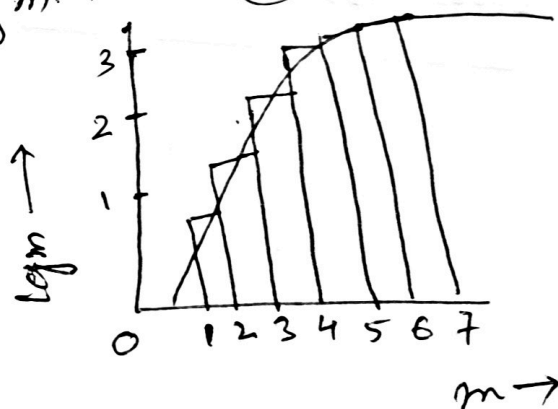
The calculation of  $n!$  becomes very laborious for large values of  $n$ . The Stirling's formula gives the approximate value of  $\log n!$ , when  $n$  is given large.

$$\log n! = n \log n - n \quad \text{--- (1)}$$

By the definition of  $n!$ , we have

$$n! = 1 \times 2 \times 3 \times \dots \times (n-1) \times n$$

$$\begin{aligned} \therefore \log n! &= \log 1 + \log 2 + \dots + \log n \\ &= \sum_{m=1}^n \log m \quad \text{--- (2)} \end{aligned}$$



Now we plot a graph with  $y = \log m$  and draw ordinates to it for values of  $m=1, 2, 3, \dots, n$ . We can approximate this sum by an integral. This replacement by an integral is increasingly good in the range where  $m$  is large, since  $\log m$  varies only slightly when  $m$  is increased by unity. With this approximation, equation (1) becomes

$$\log n! \approx \int_1^n 1 \cdot \log x \, dx \quad \text{--- (B)}$$

Integrating by parts

$$\log n! \approx (x \log x) \Big|_1^n - \int_1^n \frac{1}{x} x \, dx$$

$$\approx n \log n - n + 1$$

Neglected 1,

$$\log n! \approx n \log n - n$$

This is simplified as Stirling's theorem

General sta

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The distribution  
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