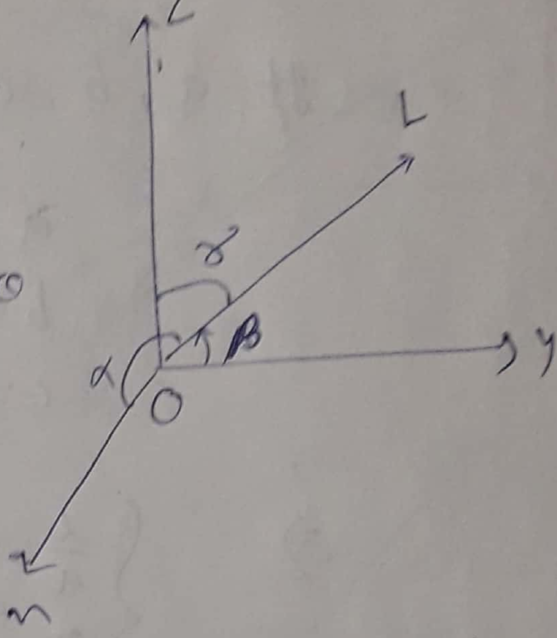
ত্ৰিমাত্ৰিক জ্যামিতি
Three Dimensional Geometry

ৰেখাৰ দিক্‌কোসাইন আৰু দিক্‌অনুপাত (Direction Cosines and Direction Ratios of a line)



দিক্‌কোসাইন: OL ৰেখাই অক্ষবোৰৰ লগত কৰা α, β আৰু γ কোণৰ কোসাইন $\cos\alpha$, $\cos\beta$ আৰু $\cos\gamma$ ক OL ৰেখাডালৰ দিক্‌কোসাইন (direction cosine) বোলে।

[If α, β, γ are the angles made by a line OL with positive ox, oy and oz, then $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called the direction cosines of the line OL.]

সাধাৰণতে (in general), আমি লিখোঁ (we write)

$$l = \cos\alpha$$
$$m = \cos\beta$$
$$n = \cos\gamma.$$

দিক্‌অনুপাত (direction ratios): → ~~দিক্‌অনুপাত~~ দিক্‌কোসাইনৰ সমানুপাতত থকা যিকোনো তিনিটা সংখ্যাক ৰেখাডালৰ দিক্‌অনুপাত বোলে।

[Any three numbers which are proportional to direction cosines of a line are called the direction ratios of the line.]

If a, b, c are d.r. of a line, then

$$\left.\begin{aligned} a &= \lambda l, \\ b &= \lambda m \\ c &= \lambda n \end{aligned}\right\} \text{①}, \quad \lambda \in \mathbb{R}, \lambda \neq 0.$$

$$\Rightarrow \left\{\begin{aligned} \frac{l}{a} &= \lambda \\ \frac{m}{b} &= \lambda \\ \frac{m}{c} &= \lambda \end{aligned}\right.$$

$$\Rightarrow \frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \lambda \longrightarrow ②$$

$$① \Rightarrow \lambda^2 l^2 + \lambda^2 m^2 + \lambda^2 n^2 = a^2 + b^2 + c^2$$

$$\Rightarrow a^2 + b^2 + c^2 = \lambda^2 (l^2 + m^2 + n^2)$$

$$= \lambda^2, \quad \because l^2 + m^2 + n^2 = 1.$$

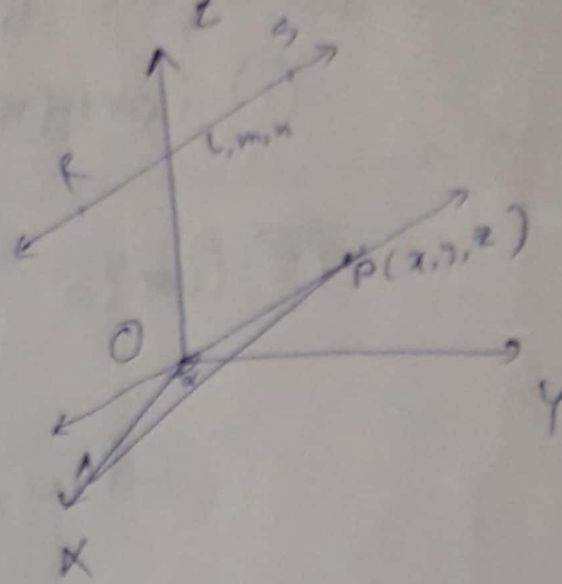
$$\Rightarrow \lambda = \pm \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$\therefore ② \Rightarrow \frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{\pm 1}{\sqrt{a^2 + b^2 + c^2}} \longrightarrow ③$$

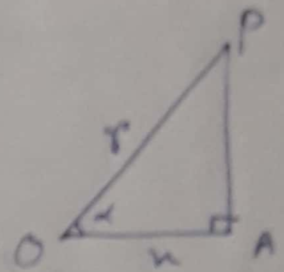
$$\Rightarrow \left\{\begin{aligned} & l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}} \\ & n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}} \end{aligned}\right.$$

8. To prove

$$\boxed{l^2+m^2+n^2=1}$$



Pf: ধরা হ'ল RS রেখার d.c. (দিক্‌কোসাইন) l, m, n. এই PS রেখার সমান্তরালকে (parallel) মূলবিন্দুতে (origin) অঙ্কনে সরল রেখা টানা হ'ল। এই রেখার উপরে P(x, y, z) একটি বিন্দু। P-থেকে x-অক্ষের উপরে PA লম্ব (⊥) টানা হ'ল। ধরো OP = r.



এখন,

$$l = \cos\alpha = \frac{OA}{OP} = \frac{x}{r}$$

$$\Rightarrow x = lr.$$

Similarly,

$$y = mr \text{ and } z = nr$$

Squaring & adding

$$x^2+y^2+z^2 = r^2(l^2+m^2+n^2) \rightarrow (1)$$

But, we know that

$$x^2+y^2+z^2 = r^2.$$

$$\therefore (1) \Rightarrow l^2+m^2+n^2 = 1.$$

Proved.

ই. $P(x_1, y_1, z_1)$ এবং $Q(x_2, y_2, z_2)$ বিন্দুর (pt.) সংযোজক সরল রেখার

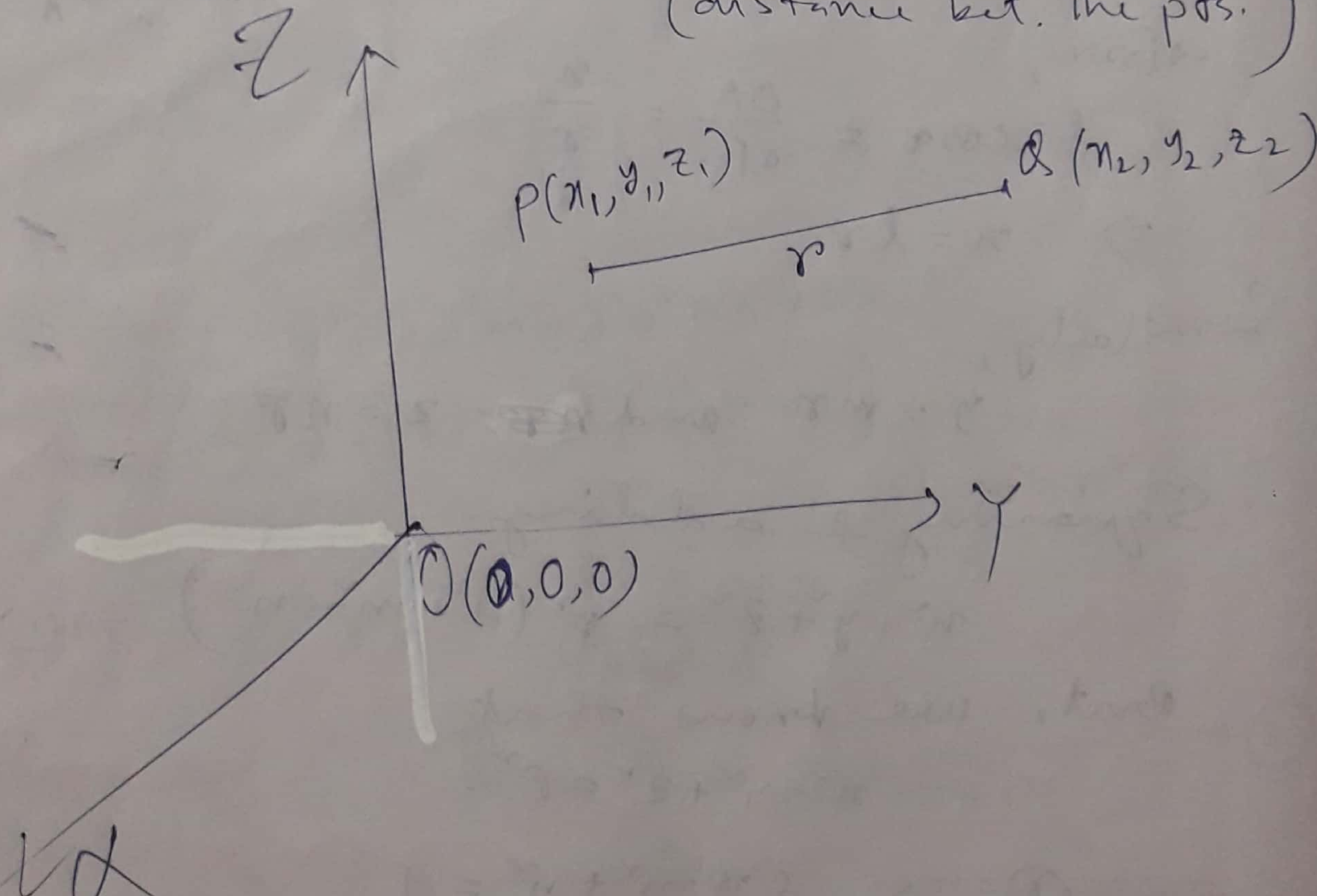
① দিকানুপাত : $x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1$

② দিকাংশ : $\dfrac{x_2 - x_1}{r},\ \dfrac{y_2 - y_1}{r},\ \dfrac{z_2 - z_1}{r}$

where,

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

= বিন্দু দুটির মধ্যে দূরত্ব।
(distance betn. the pts.)



Ex. 1. যদি এডাল ৰেখাই x, y আৰু z অক্ষৰ লগত ধনাত্মক দিশত ক্ৰমে $30°, 60°$ আৰু $90°$ কোণ সৃষ্টি কৰে তেন্তে ইয়াৰ দিশাংককেইটা নিৰ্ণয় কৰা,

(Find the d.c. of the line which makes angles $30°, 60°$ and $90°$ with the positive direction of axes.)

Solⁿ. $l = \cos 30° = \sqrt{3}/2$

$m = \cos 60° = \frac{1}{2}$

$n = \cos 90° = 0.$

$\left.\begin{array}{l} l = \sqrt{3}/2 \\ m = \frac{1}{2} \\ n = 0 \end{array}\right\} \leftarrow$ Ans

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Ex. 2. এডাল ৰেখাৰ d.r. $1, 2, -2$ হ'লে ইয়াৰ d.c. নিৰ্ণয় কৰা,

Solⁿ.

$$l = \frac{1}{\sqrt{1^2+2^2+(-2)^2}} = \frac{1}{3}$$

$$m = \frac{2}{\sqrt{1^2+2^2+(-2)^2}} = \frac{2}{3}$$

$$n = \frac{-2}{\sqrt{1^2+2^2+(-2)^2}} = \frac{-2}{3}$$

Ex. 3. Find the d.c. of the line joining $P(-2, 4, -5)$, $Q(1, 2, 3)$.

Solⁿ: $l = \frac{x_2 - x_1}{r}$, $m = \frac{y_2 - y_1}{r}$, $n = \frac{z_2 - z_1}{r}$

Here, $r = PQ = \sqrt{\{1-(-2)\}^2 + (2-4)^2 + \{3-(-5)\}^2}$

$= \sqrt{9 + 4 + 64} = \sqrt{77}$

$\therefore\ l = \frac{3}{\sqrt{77}},\ m = \frac{-2}{\sqrt{77}},\ n = \frac{8}{\sqrt{77}}$