Ex. Given $f(0) = -18$, $f(1) = 0$, $f(2) = 28$, $f(3) = 0$, $f(4) = -126$, $f(6) = 0$

Find the form of the function assuming it to be a polynomial of x.

Sol^n: Let us form the divided difference table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
0	-18					
		$\frac{0+18}{1-0} = 18$				
1	0		$\frac{28-18}{2-0} = 5$			
		$\frac{28-0}{2-1} = 28$		$\frac{-28-5}{3-0} = -11$		
2	28		$\frac{-28-28}{3-1} = -28$		$\frac{-7+11}{4-0} = 1$	
		$\frac{0-28}{3-2} = -28$		$\frac{-49+28}{4-1} = -7$		$\frac{7-1}{6-0} = 1$
3	0		$\frac{-126+28}{4-2} = -49$		$\frac{28+7}{6-1} = 7$	
		$\frac{-126-0}{4-3} = -126$		$\frac{63+49}{6-2} = 28$		
4	-126		$\frac{63+126}{6-3} = 63$			
		$\frac{0+126}{6-4} = 63$				
6	0					

Now by Newton's divided difference formula

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2)$$
$$+ \cdots + (x-x_0)(x-x_1)(x-x_2)\cdots(x-x_4) f(x_0, x_1, \ldots, x_5)$$

$$= -18 + (x-0)18 + (x-0)(x-1)5 + (x-0)(x-1)(x-2)(-11)$$
$$+ (x-0)(x-1)(x-2)(x-3)\cdot 1$$
$$+ (x-0)(x-1)(x-2)(x-3)(x-4)\cdot 1$$

$$= -18 + 18x + 5x(x-1) - 11x(x-1)(x-2)$$
$$+ x(x-1)(x-2)(x-3) + x(x-1)(x-2)\cdots(x-4)$$

Ex. The following table is given

x:	0	1	2	5
$f(x)$:	2	3	12	147

Find $f(x)$.

Sol^n: Let us first form the divided diff. table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	2			
		$\frac{3-2}{1-0} = 1$		
1	3		$\frac{9-1}{2-0} = 4$	
		$\frac{12-3}{2-1} = 9$		$\frac{9-4}{5-0} = 1$
2	12		$\frac{45-9}{5-1} = 9$	
		$\frac{147-12}{5-2} = 45$		
5	147			

Now, by Newton's divided difference formula

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3)$$

$$= 2 + (x-0).1 + (x-0)(x-1).4 + (x-0)(x-1)(x-2).1$$

$$\therefore f(x) = 2 + x + 4x(x-1) + x(x-1)(x-2).$$

Ex. Find the value of $f(2)$ and $f(8)$ from the following table.

x:	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

Sol: We form the following ~~Divided~~ difference table:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
4	48				
		$\frac{100-48}{5-4} = 52$			
5	100		$\frac{97-52}{7-4} = 15$		
		$\frac{294-100}{7-5} = 97$		$\frac{21-15}{10-4} = 1$	
7	294		$\frac{202-97}{10-5} = 21$		0
		$\frac{900-294}{10-7} = 202$		$\frac{27-21}{11-5} = 1$	0
10	900		$\frac{310-202}{11-7} = 27$		0
		$\frac{1210-900}{11-10} = 310$		$\frac{33-27}{13-7} = 1$	
11	1210		$\frac{409-310}{13-10} = 33$		
		$\frac{2028-1210}{13-11} = 409$			
13	2028				

By Newton's divided difference formula

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3)$$

$$\therefore f(2) = 48 + (2-4)\times 52 + (2-4)(2-5)\times 15 + (2-4)(2-5)(2-7)\times 1.$$

$$\Rightarrow f(2) = 48 + (-104) + 90 - 30 = 228 - 164 = 164$$

$$= 4 \quad //$$

Again,

$$f(8) = 48 + (8-4)\times 52 + (8-4)(8-5)\times 15 + (8-4)(8-5)(8-7)\times 1$$

$$\Rightarrow f(8) = 448. \quad //$$
