§. Newton's d.d Interpolation formula:

(Theorem:- The 'n'th divided difference of a pollynomial of degree 'n' is constant and higher order differences are zero.)

Newton's D.D. Formula:-

Let $f(x_0), f(x_1), f(x_2), \ldots, f(x_n)$ be the $(n+1)$ values of the function $f(x)$ corresponding to the values of $x_0, x_1, x_2, \ldots, x_n$ of x.

Now, the 1st d.d. of $f(x)$ for the arguments x and x_0 is

$$\delta(x, x_0) = f(x, x_0) = \frac{f(x) - f(x_0)}{x - x_0}$$

$$\Rightarrow f(x) = f(x_0) + (x - x_0) f(x, x_0) \longrightarrow (i)$$

Next,

$$f(x, x_0, x_1) = \frac{f(x, x_0) - f(x_0, x_1)}{x - x_1}$$

$$\Rightarrow f(x, x_0) = f(x_0, x_1) + (x - x_1) f(x, x_0, x_1)$$

$$(i) \Rightarrow f(x) = f(x_0) + (x - x_0)\left[f(x_0, x_1) + (x - x_1) f(x, x_0, x_1)\right]$$

$$= f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x, x_0, x_1) \longrightarrow (ii)$$

Again,

$$f(x, x_0, x_1, x_2) = \frac{f(x, x_0, x_1) - f(x_0, x_1, x_2)}{x - x_2}$$

$$\Rightarrow f(x, x_0, x_1) = f(x_0, x_1, x_2) + (x - x_2) f(x, x_0, x_1, x_2)$$

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$(ii) \Rightarrow f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1)$

$\cdot [f(x_0, x_1, x_2) + (x-x_2) f(x, x_0, x_1, x_2)]$

$\Rightarrow f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1)$

$\cdot f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x, x_0, x_1, x_2)$

Proceeding in this way, until the last argument x_n comes, we get,

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2)$$
$$+ (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3)$$
$$+ \cdots + (x-x_0)(x-x_1)\cdots(x-x_{n-1}) f(x_0, x_1, \cdots, x_n)$$
$$+ (x-x_0)(x-x_1)\cdots\cdots(x-x_n) f(x, x_0, x_1, \cdots x_n)$$

or, $f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2)$

$+ \cdots + (x-x_0)(x-x_1)\cdots\cdots(x-x_{n-1}) f(x_0, x_1, \cdots x_n)$

$+ R_n \longrightarrow (iii)$

where, $R_n = (x-x_0)(x-x_1)\cdots(x-x_n) f(x, x_0, x_1, \cdots x_n)$

If we consider $f(x)$ to be a pollynomial of degree 'n' then the n-th ~~didio~~ divided difference $f(x_0, x_1, \cdots x_n)$ is constant and the $(n+1)$'th d.d. $f(x, x_0, x_1, \cdots x_n)$ is zero.

$\therefore R_n = 0.$

$\because (iii) \Rightarrow f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2)$

$+ \cdots + (x-x_0)(x-x_1)\cdots(x-x_{n-1}) f(x_0, x_1, \cdots x_n)$

which is clearly a polly of degree 'n'.