

Definition: Divided Difference:

Let $f(x_0), f(x_1), \dots, f(x_n)$ be the values of the function $f(x)$ corresponding to the values $x_0, x_1, x_2, \dots, x_n$ of x , where $x_1 - x_0, x_2 - x_1, x_3 - x_2, \dots, x_n - x_{n-1}$ are not necessarily equal.

Then first divided difference (d.d.) of $f(x)$ for the arguments x_0 and x_1 denoted by $\Delta_{x_1} f(x_0)$ or $f(x_0, x_1)$ is defined by

$$f(x_0, x_1) = \frac{f(x_0) - f(x_1)}{x_0 - x_1} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

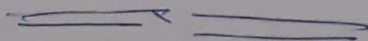
The 2nd divided difference (2nd d.d.) of $f(x)$ for the arguments x_0, x_1 and x_2 denoted by $\Delta_{x_1 x_2}^2 f(x_0)$ or $f(x_0, x_1, x_2)$ is

defined as

$$f(x_0, x_1, x_2) = \frac{f(x_0, x_1) - f(x_1, x_2)}{x_0 - x_2}$$

Similarly, n th divided difference for the arguments $x_0, x_1, x_2, \dots, x_n$ is defined as

$$f(x_0, x_1, \dots, x_n) = \frac{f(x_0, x_1, \dots, x_{n-1}) - f(x_1, x_2, \dots, x_n)}{x_0 - x_n}$$



Formation of divided difference table :-

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
x_0	$f(x_0)$	$f(x_0, x_1)$	$f(x_0, x_1, x_2)$	$f(x_0, x_1, x_2, x_3)$	$f(x_0, x_1, x_2, x_3, x_4)$
x_1	$f(x_1)$	$f(x_1, x_2)$	$f(x_1, x_2, x_3)$	$f(x_1, x_2, x_3, x_4)$	
x_2	$f(x_2)$	$f(x_2, x_3)$	$f(x_2, x_3, x_4)$		
x_3	$f(x_3)$				
x_4	$f(x_4)$				

Ex: Form a d.d. for the following data :

x	2	4	9	10
$f(x)$	4	56	711	980

Solⁿ

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
2	4			
4	56	$\frac{56-4}{4-2} = 26$	$\frac{131-26}{9-2} = 15$	
9	711	$\frac{711-56}{9-4} = 131$		$\frac{23-15}{10-2} = 1$
10	980	$\frac{980-711}{10-9} = 269$	$\frac{269-131}{10-4} = 23$	

Ex. Find the first three d.d. for the function $\frac{1}{n^2}$ for the arguments a, b, c, d .

Sol: Let $f(n) = \frac{1}{n^2}$.

$$\therefore \text{1st. d.d.} = f(a, b) = \frac{\frac{1}{b^2} - \frac{1}{a^2}}{b - a}$$

$$= \frac{a^2 - b^2}{b^2 a^2 (b - a)} = - \frac{a + b}{a^2 b^2}, \quad a \neq b.$$

$$f(a, b, c) = \frac{f(b, c) - f(a, b)}{c - a}$$

$$= \frac{-\frac{b+c}{b^2 c^2} + \frac{a+b}{a^2 b^2}}{c - a} = \frac{-a^2(b+c) + c^2(a+b)}{a^2 b^2 c^2 (c - a)}$$

$$= \frac{ac(c-a) + b(c^2 - a^2)}{a^2 b^2 c^2 (c - a)} = \frac{(c-a)(ab+bc+ca)}{a^2 b^2 c^2 (c-a)}$$

$$= \frac{ab+bc+ca}{a^2 b^2 c^2}$$

$$f(a, b, c, d) = \frac{f(b, c, d) - f(a, b, c)}{d - a}$$

$$= \frac{\frac{bc+cd+db}{b^2 c^2 d^2} - \frac{ab+bc+ca}{a^2 b^2 c^2}}{d - a}$$

$$= \frac{a^2 bc + a^2 cd + a^2 bd - d^2 ab - d^2 bc - d^2 ca}{a^2 b^2 c^2 d^2 (d - a)}$$

$$= \frac{-dab(d-a) - bc(d^2 - a^2) - dea(d-a)}{a^2 b^2 c^2 d^2 (d-a)}$$

$$= \frac{(d-a)[-dab - bcd - bca - dea]}{a^2 b^2 c^2 d^2 (d-a)}$$

$$= - \frac{abc + bcd + cda + dab}{a^2 b^2 c^2 d^2}$$

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