(*) Entropy change with Temperature :-

Since S is a state function, we may write,

$$S = f(V, T)$$

$$\Rightarrow dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

at Const V, $dV = 0$ ∴

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT$$

$$= \frac{1}{T} \cdot \left(\frac{T\partial S}{\partial T}\right)_V dT$$

$$\frac{dq}{T} = dS$$

$$\left(\frac{dq}{dT}\right)_V = C_V$$

$$= \frac{1}{T} \cdot C_V \cdot dT$$

$$= \frac{C_V}{T} dT$$

Integrating,

$$\int_1^2 ds = \int_{T_1}^{T_2} c_v \frac{dT}{T}$$

$$\Rightarrow \Delta S_v = C_v \ln \frac{T_2}{T_1}$$

#

Similarly at const. Press^r,

$$\Delta S_p = C_p \ln \frac{T_2}{T_1}$$

#

Note!

$\because S = f(P, T)$

$$ds = \left(\frac{\partial s}{\partial T}\right)_P dT + \left(\frac{\partial s}{\partial P}\right)_T dP$$

at const P, $dP = 0$

$$\therefore ds = \left(\frac{\partial s}{\partial T}\right)_P dT$$

$$= \frac{1}{T}\left(\frac{T \cdot \partial s}{\partial T}\right)_P dT$$

$$= \frac{1}{T} C_p \cdot dT$$

$$\Rightarrow ds = C_p \frac{dT}{T}$$

integrating,

$$\int_1^2 ds = \int_{T_1}^{T_2} C_p \frac{dT}{T}$$

$$\Delta S_p = C_p \ln \frac{T_2}{T_1}$$

*

* Entropy of an ideal gas :-

From the first law of thermodynamics,

$$dq = dU + PdV$$

Dividing both side by T

$$\frac{dq}{T} = \frac{dU}{T} + \frac{PdV}{T}$$

$$= C_V \frac{dT}{T} + R\frac{dV}{V}$$

$$\Rightarrow ds = C_V \, d\ln T + R \, d\ln V$$

for $n = 1$, ideal gas, $PV = RT$

$P = RT/V$

$* \left(\frac{dU}{dT}\right)_V = C_V$

$dU = C_V dT$

Integrating, $S = C_V \ln T + R \ln V + S_0 \rightarrow (1)$

Where S_0 is constant; heat capacities assumed to remain unaltered. Since, $C_p - C_V = R$

so,

$$S = (C_p - R)\ln T + R \ln \frac{RT}{P} + S_0$$

$$= C_p \ln T - R \ln P + S_0' \rightarrow (2)$$

It must be noted that, the value of the constant S_0 and S_0' can't be obtained from the thermal capacities or the parameters alone. As such, we can't ascertain the absolute value of the entropy of a perfect gas from these relation. But the changes in entropy can be easily determined. Thus, if the temp of a gas changes from T_1 to T_2 and from vol^m V_1 to V_2 then for one gm mole of the gas,

$$S_1 = C_v \ln T_1 + R \ln V_1 + S_0$$

$$S_2 = C_v \ln T_2 + R \ln V_2 + S_0$$

$$\Delta S = S_2 - S_1$$

$$= C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

If the change occurs under isothermal cond^n ie. T is const. then

$$\Delta S = R \ln \frac{V_2}{V_1} =$$

Similarly, $\Delta S = R \ln \frac{P_1}{P_2}$

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