Clausius Inequality :—

Since S is a state function. If the cyclic Process involves going from A to B and then Coming back to A, then

$$\oint ds = \int_A^B ds + \int_B^A ds = 0$$

$$\therefore \int_A^B ds = -\int_B^A ds \longrightarrow (1)$$

ie. the entropy change in going from B to A is equal but opposite in sign to that of A to B.

If we assume that the surroundings always transfer heat reversibly, then the entropy change in any process carried out reversibly will be the –ve of the entropy change of the surrounding, i.e.

$$\Delta S_{sys} = -\Delta S_{sur}.$$

$$\Rightarrow \Delta S_{Total} = \Delta S_{sys} + \Delta S_{surr} = 0 \rightarrow (2)$$

Thus the sum of entropy changes for the system and of the surrounding will always be zero. Hence, it may be concluded that in reversible processes entropy is merely transfered betn the system and surroundings and that the total entropy changes is zero.

Now let us consider the process $A \rightarrow B$ is irreversible, we can assume that the reverse process $B \rightarrow A$ is carried out reversibly. Then we may have,

$$\begin{tikzcd} A \arrow[r, "dq_1^*", "\text{Irreversible}"'] & B \arrow[d] \\ \cdot \arrow[u] & \cdot \arrow[l, "dq_2"', "\text{reversible}"] \end{tikzcd}$$

Where,
dq_1^* is heat change irreversibly
dq_2 is heat change reversibly in the cycle.

Now,

$$\oint \frac{dq_1^*}{T} = \int_A^B \frac{dq_1^*}{T} + \int_B^A \frac{dq_2}{T}$$

From Eqn. (1) ⇒

$$\oint \frac{dq_1^*}{T} = \int_A^B \frac{dq_1^*}{T} - \int_A^B \frac{dq}{T}$$

We know, $\oint \frac{dq_1^*}{T} < 0$

$$\therefore \int_A^B \frac{dq_1^*}{T} < \int_A^B \frac{dq}{T}$$

$$\Rightarrow \int_A^B \frac{dq_1^*}{T} < \int_A^B ds$$

$$\Rightarrow \int_A^B \frac{dq_1^*}{T} < \Delta S_{AB} \longrightarrow (3)$$

This expression is known as Clausius's inequality.

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