Problem Related Newton's backward int. formula

[Note: (i) Newton's forward interpolation formula is used mainly for interpolating the values of y near the begining of a set of tabular values.

(ii) Newton's backward interpolation formula is mainly used for interpolating the values of y near the end of a set of tabular values.]

Ex. Given

x:	1	2	3	4	5	6	7	8
$f(x)$:	1	8	27	64	125	216	343	512

Find $f(7.5)$.

Sol^n^ Obviously, the value to be interpolated lies at the end of the given observations i.e. near $x=8$. Hence, in this case, we use Newton's backward interpolation formula.

Here,

$$u = \frac{x-(a+nh)}{h}$$

$$= \frac{7.5-(1+7\times 1)}{1}$$

$$= 7.5-8$$

$$= -0.5$$

$x = 7.5$

$a = 1, h = 1$

$n = 7$

To calculate Backward Difference, we construct the following difference table :

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
1	1			
		7		
2	8		12	
		19		6
3	27		18	
		37		6
4	64		24	
		61		6
5	125		30	
		91		6
6	216		36	
		127		6
7	343		42	
		169		
8	512			

[$\because$ $\nabla^3 f(x)$ is constant, we leave higher order differences]

Now, by Newton's backward interpolation formula, we have

$$f(a+nh+uh) = f(a+nh) + u\nabla f(a+nh)$$
$$+ \frac{u(u+1)}{\lfloor 2}\nabla^2 f(a+nh) + \frac{u(u+1)(u+2)}{\lfloor 3}\nabla^3 f(a+nh)$$

$$\Rightarrow f(7.5) = f(8) + (-0.5)\nabla f(8) + \frac{(-0.5)(-0.5+1)}{2}\nabla^2 f(8) + \frac{(-0.5)(-0.5+1)(-0.5+2)}{6}\nabla^3 f(8)$$

$$= 512 + (-0.5)\times 169 + \frac{(-0.5)(0.5)}{2}\times 42 + \frac{(-0.5)(0.5)(1.5)}{6}\times 6$$

$$= 512 - 84.5 - 5.25 - 0.375$$

$$= 421.875 \longleftarrow \text{Ans.}$$