Miller indices:-

In order to specify a crystal miler introduced a set of three numbers. This set of three numbers is called miller indices of the plane.

The main properties of miller indices is that ~~that~~ they are same for all parallel planes. The method of finding the miller indices is as follows.

(i) We first find the intercepts of the plane along x, y and z axes in terms of the lattice constants a, b and c respectively.

(ii) Then we find the reciprocals of these numbers

(iii) Next determine the least common denominator (L.C.D) and multiply each by this L.C.D. The obtained results may be given in the form h, k, and l and are known as miller indices of the concerned plane.

Notes:- (1) If a plane is parallel to any of three axes, its intercept on such axis is infinity. Thus its Miller indices is $\frac{1}{\infty}$ or 0

(2) When a plane cuts an axis on the negative side of the origing the corresponding index is negative,

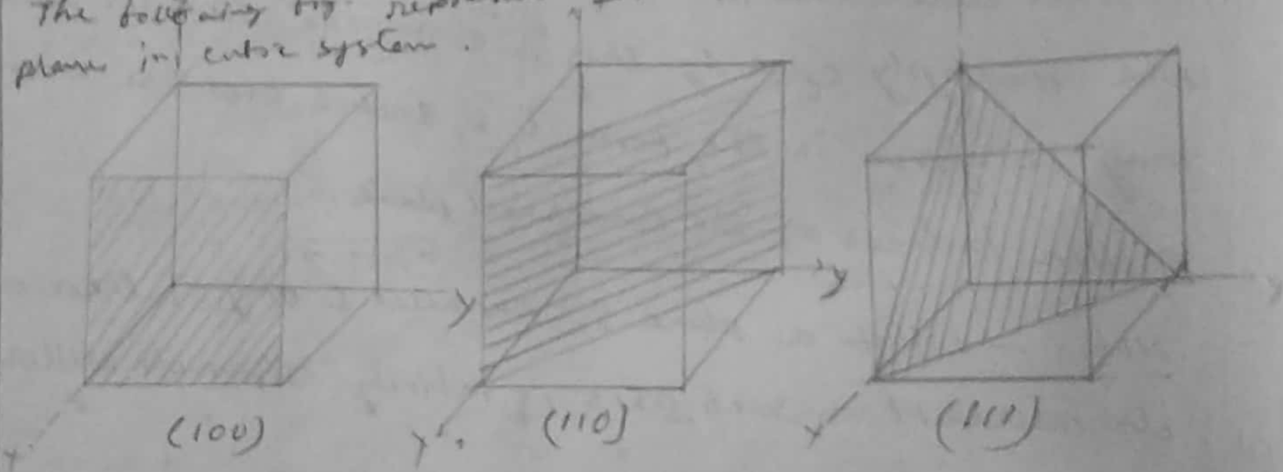
Example! If a plane have intercepts 4, 1, and 2 on the axes x, y and z respectively, Then the reciprocals are $\frac{1}{4}$, 1 and $\frac{1}{2}$. In this case L C D is 4. Now multiplying the each reciprocals by 4, then we obtain 1, 4 and 2, Thus (1, 4, 2) are the Miller indices of the plane.

It may be noted that miller indices h, k, l do not explain any single plane but a group of parallel planes. Thus all the parallel planes have same miller indices (h, k, l). A plane (nh, nk, nl) is the same as (h, k, l) plane. Hence the plane (6, 6, 6) is the same as (2, 2, 2) plane or (1, 1, 1) plane etc.

Orientation of a plane by miller indices -

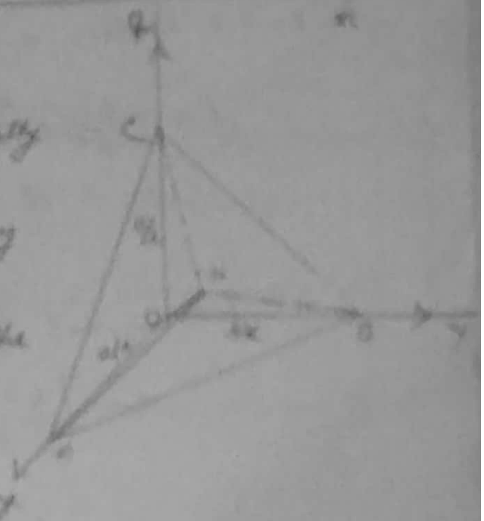
The miller indices (h, k, l) may denote a single plane or a set of parallel planes. If a plane cuts an axis on the negative side of the origine, the index is negative and denoted as $(h \bar{k} l)$ by placing a minus (-) sign above the corresponding index (in the case of $\bar{k}$). For an intercept at infinity the corresponding index is zero. For the crystal plane having intercept 2a, 3b and c on the crystal axes $\vec{a}$, $\vec{b}$ and $\vec{c}$, the reciprocals of the 2, 3 and 1 are $\frac{1}{2}$, $\frac{1}{3}$ and 1. The L.C.M of these are 6. Therefore the miller indices are (3, 2, 6). Hence the plane refered to as (3, 2, 6) plane.

The following fig represents the miller indices of particular plane in cubic system.



The six faces of a cubical crystal are (100), (010) (001) $(\bar{1}00)$ $(0\bar{1}0)$, $(00\bar{1})$.

Inter planar Distance OR lattice plane:-



Let the axes OX, OY and OZ are mutually perpendicular to each other. Let the origin O is taken at any lattice point. Assume any set of crystal plane having miller indices (h, k, l). Let the reference plane passes through the origin and next plane cuts the intercepts $\frac{a}{h}$, $\frac{b}{k}$ and $\frac{c}{l}$ on X, Y and Z axes respectively. Let us draw perpendicular ON from origin to the plane ABC. Let $ON = d$ = perpendicular distance from origin to the plane. = distance between adjacent planes (i.e. interplanar distance)

If ON has direction cosines $\cos\alpha$, $\cos\beta$ and $\cos\gamma$

then
$$\left.\begin{aligned} d &= \frac{a}{h}\cos\alpha \\ d &= \frac{b}{k}\cos\beta \\ d &= \frac{c}{l}\cos\gamma \end{aligned}\right\} \text{——— (1)}$$

where $\alpha = \angle NOA$, $\beta = \angle NOB$ and $\gamma = \angle NOC$.

According to law of direction cosines

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad \text{——— (2)}$$

From (1) and (2) we have

$$d^2\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right) = 1$$

$$\text{or } d^2 = \frac{1}{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}$$

$$\therefore d = \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right)^{-\frac{1}{2}} \quad \text{——— (3)}$$

... expression for the interplanar distance.

In cubic system $a = b = c$, we get

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}} \quad \text{---} (4)$$

For Tetragonal system $a = b$, hence

$$d = \left(\frac{h^2+k^2}{a^2} + \frac{l^2}{c^2}\right)^{-1/2} \quad \text{---} (5)$$

For Simple cubic crystal

$$d_{100} = \frac{a}{\sqrt{1^2+0^2+0^2}} = a$$

$\therefore d_{100} = a$.

$d_{110} = \frac{a}{\sqrt{2}}$ and $d_{111} = \frac{a}{\sqrt{3}}$

Hence the separation between successive (100) (110 and (111) planes are a, $\frac{a}{\sqrt{2}}$ and $\frac{a}{\sqrt{3}}$ respectively. Hence

$$d_{100} : d_{110} : d_{111} = 1 : \frac{1}{\sqrt{2}} : \frac{1}{\sqrt{3}}$$

Q Show that for a simple cubic crystal

$$d_{100} : d_{110} : d_{111} = \sqrt{6} : \sqrt{3} : \sqrt{2}$$

Solⁿ If a be the cube edge element the distance betⁿ adjacent planes is

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

From this relation we get

$$d_{100} = \frac{a}{\sqrt{1^2+0+0}} = a$$

$$d_{110} = \frac{a}{\sqrt{1^2+1^2+0}} = \frac{a}{\sqrt{2}}$$

$$d_{111} = \frac{a}{\sqrt{1^2+1^2+1^2}} = \frac{a}{\sqrt{3}}$$

Hence $d_{100} : d_{110} : d_{111}$

$$= a : \frac{a}{\sqrt{2}} : \frac{a}{\sqrt{3}}$$

$$= \sqrt{6} : \sqrt{3} : \sqrt{2}$$