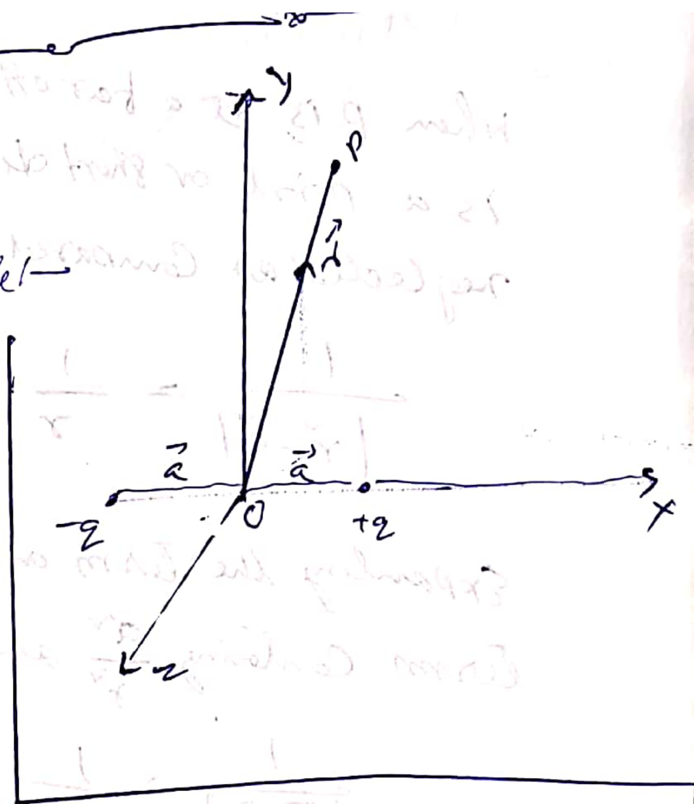
Electric dipole :-

Electric potential due to dipole :-



Consider a dipole consisting of charges $+q$ and $-q$ separated by a small distance $2\vec{a}$, lying along x-axis with its centre at the origin.

~~then~~ Dipole moment of the electric dipole $\vec{p} = 2q\vec{a}$

Consider a point p having a position vector $\vec{r}$,

potential at p due to the charge $+q$:

$$V_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{|\vec{r}-\vec{a}|}$$

and potential at p due to the charge $-q$

$$V_- = \frac{1}{4\pi\epsilon_0}\frac{q}{|\vec{r}+\vec{a}|}$$

Net potential at P due to electric dipole.

$$V = V_+ + V_-$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{|\vec{r}-\vec{a}|} + \frac{1}{|\vec{r}+\vec{a}|}\right] \quad \text{---} \rightarrow (1)$$

To find the value $|\vec{r}-\vec{a}|$, consider the term

$$|\vec{r}-\vec{a}|^2 = (\vec{r}-\vec{a})(\vec{r}-\vec{a})$$

$$= r^2 + a^2 - 2\vec{r}.\vec{a}$$

$$= r^2\left[1 + \frac{a^2}{r^2} - 2\frac{\vec{r}.\vec{a}}{r^2}\right]$$

$$\therefore |\vec{r}-\vec{a}| = r\left[1 + \frac{a^2}{r^2} - 2\frac{\vec{r}.\vec{a}}{r^2}\right]^{1/2}$$

or $$\frac{1}{|\vec{r}-\vec{a}|} = \frac{1}{r}\left[1 + \frac{a^2}{r^2} - 2\frac{\vec{r}.\vec{a}}{r^2}\right]^{-1/2}$$

When P is at a far off point from O or the dipole is a point or short dipole i.e. $r > a$, $\frac{a^2}{r^2}$ can be neglected as compared to a. Hence

$$\frac{1}{|\vec{r}-\vec{a}|} = \frac{1}{r}\left[1 - \frac{2\vec{r}.\vec{a}}{r^2}\right]^{-1/2}$$

Expanding the term on R.H. side and neglecting the terms containing $\frac{a^2}{r^2}$ and higher powers we get

$$\frac{1}{|\vec{r}-\vec{a}|} = \frac{1}{r}\left[1 + \frac{\vec{r}.\vec{a}}{r^2}\right] = \frac{1}{r} + \frac{\vec{r}.\vec{a}}{r^3}$$

Similarly $$\frac{1}{|\vec{r}+\vec{a}|} = \frac{1}{r} - \frac{\vec{r}.\vec{a}}{r^3}$$

Substituting these values in equ^n (1) we get

$\rightarrow$

$$V = \frac{2q}{4\pi\epsilon_0}\frac{\vec{r}\cdot\vec{a}}{r^3} = \frac{\vec{r}\cdot 2q\vec{a}}{4\pi\epsilon_0 r^3} = \frac{\vec{r}\cdot\vec{p}}{4\pi\epsilon_0 r^3} = \frac{1}{4\pi\epsilon_0}\vec{p}\left[\frac{\vec{r}}{r^3}\right]$$

or $V = -\frac{1}{4\pi\epsilon_0}\vec{p}\cdot\nabla\left(\frac{1}{r}\right)$ $\boxed{\because \nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}}$

Also $V = \frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3} = \frac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_0 r^2}$ ——— (2)

Where $\hat{r}$ is a unit vector in the direction of $\vec{r}$.

If θ is the angle between $\vec{p}$ and $\vec{r}$, then

$$V = \frac{pr\cos\theta}{4\pi\epsilon_0 r^3} = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} \text{ ——— (3)}$$

Thus we find that potential at a point due to a short dipole varies inversely as the square of the distance.

(i) potential on the axial line

When the point p lies on the axis of the dipole (or axial line) the vectors $\vec{p}$ and $\vec{r}$ are in the same direction i.e $\theta = 0$ and $\cos\theta = 1$ i.e along x-axis.

$\therefore \vec{p}\cdot\vec{r} = pr$

substituting in eqn (2), we have

$$V = \frac{pr}{4\pi\epsilon_0 r^3} = \frac{p}{4\pi\epsilon_0 r^2}$$

When the point p lies on the axial line of the dipole in a direction of opposite to that of $\vec{p}$, $\theta = \pi$ and $\cos\theta = -1$

$$\therefore V = -\frac{p}{4\pi\epsilon_0 r^2}$$

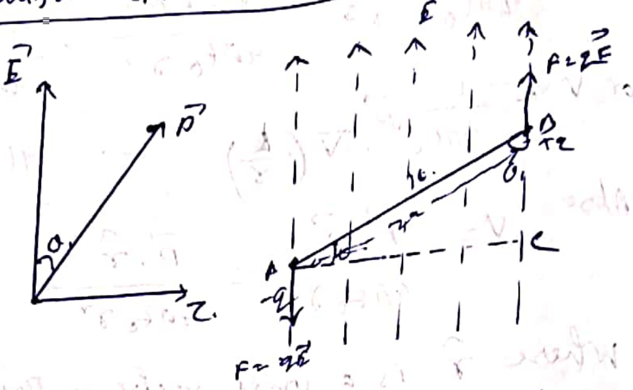
(ii) potential on the normal to the axis. (Equatorial line)

When the point p lies on the normal to the axis $\vec{p}$ and $\vec{r}$ are at right angle to each other ($\vec{p}$ along x axis and $\vec{r}$ along y-axis) or $\theta = \pi/2$ and $\cos\theta = 0$

$\therefore \vec{p}\cdot\vec{r} = 0$

$\therefore V = 0$ //

Electric dipole in an uniform external electric field.



Consider an electric dipole consisting of two equal and opposite point charges $-q$ and $+q$ separated by a small distance $AB = 2a$. having dipole moment $\vec{p} = q \times 2a$. Let the dipole be held in a uniform external electric field $\vec{E}$ at an angle θ with the direction of $\vec{E}$.

Force on charge $+q$ at B $F = q\vec{E}$ along the direction of $\vec{E}$.

Force on the charge $-q$ at A $F = -q\vec{E}$ along the direction opposite to $\vec{E}$.

Since $\vec{E}$ is uniform, therefore net force on the dipole is zero $(qE - qE) = 0$. However, as the forces are equal and opposite and parallel. acting at different point, therefore they form a couple. which rotates the dipole in the anticlockwise direction as shown in the fig, the couple tends to align the dipole axis along the direction of field $\vec{E}$.

~~Draw~~ ~~$\vec{E}$~~.

Draw $AC \perp \vec{E}$.

∴ perpendicular distance between the forces = arm of the couple.

As ~~Torque~~ = moment of the couple

τ = force × arm of the couple

$\tau = F \times AC$

or $\tau = F \times AB\sin\theta = F \times 2a\sin\theta = qE \times 2a\sin\theta$.

or $\tau = \vec{p}E\sin\theta$. ——— (1)

In the vector form.

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}} \quad \text{——— (2)}$$

The direction of $\vec{\tau}$ is given by right hand screw rule and is perpendicular to $\vec{p}$ and $\vec{E}$ i.e perpendicular to the plane of the paper.

Special case:— When $\vec{p}$ is along $\vec{E}$, $\theta = 0 \therefore \tau = PE\sin 0 = 0$

The dipole is in stable equilibrium.

When the dipole is held in a direction opposit to $\vec{E}$, the torque would turn the dipole through 180, As such the dipole will be in an unstable equilibrium.

The torque will be maximum when $\theta = 90^\circ$.

maximum value of torque $\tau = PE\sin 90 = PE$,

The unit of τ is N-m, and its dimensional formula is $[M^1L^2T^{-2}]$

potential Energy of a dipole in a uniform electric field.

The potential energy of a dipole in an electric field is defined as the amount of work done against the electric field in bringing the dipole from infinity and placing it in the desired orientation in the field.

When the dipole is placed perpendicular to the direction of electric field the work done = 0, The equipotential lines are always at right angles to the lines of electric field, Therefore the work done in bringing the charge +q and the charge -q to the same potential being equal and opposit cancelled each other. Therefore work is don only in rotating the

the dipole from the position perpendicular to the field to any other position.

Let the dipole make an angle θ with the field $\vec{E}$, then the magnitude of the couple (torque) acting on it $= PE\sin\theta$. Work done in turning the dipole from the position making an angle θ with the field to the position making an angle $\theta + d\theta$ is given by

$$dw = PE\sin\theta\, d\theta.$$

$\therefore$ potential energy of the dipole in the orientation θ

$$U = \int_{\theta=\pi/2}^{\theta=\theta} dw = \int_{\theta=\pi/2}^{\theta=\theta} PE\sin\theta\, d\theta = \left[-PE\cos\theta\right]_{\pi/2}^{\theta}.$$

$$\text{or } U = -PE\cos\theta = -\vec{P}\cdot\vec{E}$$

as $\vec{P}$ and $\vec{E}$ are vector quantities.

Minimum energy of dipole :- As explained above no work is done in placing the dipole in a direction perpendicular to the electric field. Work is only done in rotating the dipole from the position perpendicular to the field to any other position. Thus the energy of a dipole is minimum (zero) when placed at right angles to the direction of the electric field.

What is electric dipole? What is dipole moment? Calculate the electric field due to a dipole at a point (i) on the axial line (ii) on the equatorial line.

Show that the value of electric field due to a small dipole at a far off point on the axial line is twice that of an equatorial line.

equal and opposit