Ex. 10.4

Q. No (4). Show that
$(\vec{a}-\vec{b}) \times (\vec{a}+\vec{b}) = 2(\vec{a} \times \vec{b})$

Soln. L.H.S. $= (\vec{a}-\vec{b}) \times (\vec{a}+\vec{b})$

$= \vec{a} \times (\vec{a}+\vec{b}) - \vec{b} \times (\vec{a}+\vec{b})$

$= \vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}$

$= 0 + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} - 0$

$= 2(\vec{a} \times \vec{b})$

$=$ R.H.S.

showed.

Q. (5) If $(2i+6j+27k) \times (i+\lambda j+\mu k) = \vec{0}$,
then find λ and μ.

Soln. Given
$(2i+6j+27k) \times (i+\lambda j+\mu k) = \vec{0}$

$\Rightarrow \begin{vmatrix} i & j & k \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = \vec{0}$

$\Rightarrow i(6\mu - 27\lambda) - j(2\mu - 27) + k(2\lambda - 6) = 0i + 0j + 0k$

$$\Rightarrow \begin{cases} 6\mu - 27\lambda = 0 \longrightarrow (i) \\ 27 - 2\mu = 0 \longrightarrow (ii) \\ 2\lambda - 6 = 0 \longrightarrow (iii) \end{cases}$$

Now,

(iii) $\Rightarrow 2\lambda = 6$

$\Rightarrow \lambda = 3.$ ← Ans.

(ii) $\Rightarrow 2\mu = 27$

$\Rightarrow \mu = \frac{27}{2}$ ← Ans.

Putting these values of λ and μ in (1),

$\cancel{6}^{3} \cdot \frac{27}{\cancel{2}} - 27 \cdot 3 = 0$

$\Rightarrow 54 - 54 = 0$

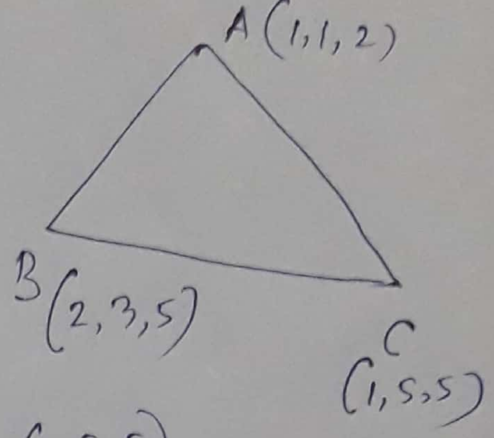
$\Rightarrow 0 = 0$, which is true.

Hence,

$$\left.\begin{aligned} \lambda &= 3 \\ \mu &= 27/2 \end{aligned}\right\} \longleftarrow \text{Ans.}$$

x

Q. No. 9 এটা সামান্তৰিকৰ শীৰ্ষবিন্দুৰ স্থানাংক $A(1,1,2)$, $B(2,3,5)$ আৰু $C(1,5,5)$। ত্ৰিভুজটোৰ কালি উলিওৱা।



Soln. Given that (দিয়া আছে)

virtices of the triange are
(ত্ৰিভুজটোৰ শীৰ্ষ বিন্দু(বোৰৰ স্থানাংক)

$A(1,1,2)$, $B(2,3,5)$ and $C(1,5,5)$.

$$\therefore \overrightarrow{AB} = (2-1)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k}$$
$$= \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overrightarrow{AC} = (1-1)\hat{i} + (5-1)\hat{j} + (5-2)\hat{k}$$
$$= 0\hat{i} + 4\hat{j} + 3\hat{k} = 4\hat{j} + 3\hat{k}.$$

$$\therefore \triangle ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$$

$$\text{Now, } \left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$$= i(6-12) - j(3-0) + k(4-0)$$
$$= -6i - 3j + 4k$$

$$\therefore \left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{61}$$

$$\therefore \triangle ABC = \frac{1}{2}\left(\sqrt{61}\right) \text{ sq. unit}$$ (বৰ্গ একক). ← Ans.