Exercise 10.4

Q. ① यदि (If) $\vec{a} = i - 7j + 7k$ तथा $\vec{b} = 3i - 2j + 2k$, तो (then) $|\vec{a} \times \vec{b}|$ ज्ञात कीजिए।

Sol^n: Given,

$$\vec{a} = i - 7j + 7k$$

$$\vec{b} = 3i - 2j + 2k$$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= i\{(-7)\cdot 2 - 7\cdot(-2)\} - j\{1\cdot 2 - 3\cdot 7\} + k(1\cdot 2 - 3\cdot$$

$$= 0.i + 19j + 19k.$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{19^2 + 19^2}$$

$$= \sqrt{19^2(1+1)}$$

$$= 19\sqrt{2} \leftarrow \text{Ans.}$$

8(2) $\vec{a}+\vec{b}$ এবং $\vec{a}-\vec{b}$ ভেক্টর দুটির প্রত্যেকটির সাথে লম্ব (orthogonal) একক ভেক্টর নির্ণয় কর, যখন $\vec{a}=3i+2j+2k$ এবং $\vec{b}=i+2j-2k$.

Sol: Given, $\vec{a}=3i+2j+2k$

& $\vec{b}=i+2j-2k$.

$\therefore \vec{a}+\vec{b} = 4i+4j$

& $\vec{a}-\vec{b} = 2i+4k$

$\therefore$ A vector $\perp^r$ to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is

$$= (\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = \begin{vmatrix} i & j & k \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

$$= i(4.4-0.0)-j(4.4-0.2)+k(4.0-4.2)$$

$$= 16i-16j-8k \quad (=\vec{c}, \text{ let})$$

Now, $|\vec{c}| = \sqrt{16^2+(-16)^2+(-8)^2}$

$$= \sqrt{8^2(2^2+2^2+1)} = 8\times 3 = 24$$

$\therefore$ Required unit vector (নির্ণেয় একক ভেক্টর) $= \dfrac{\vec{c}}{|\vec{c}|} = \dfrac{16i-16j-8k}{24} = \dfrac{2}{3}i-\dfrac{2}{3}j-\dfrac{1}{3}k$ ← Ans.

Note: Another unit vector will be $= -\left(\dfrac{2}{3}i-\dfrac{2}{3}j-\dfrac{1}{3}k\right)$

Ex. 10.4.

Q. 3. $\hat{a}$ এর প্রত্যেক উপাংশ যথাক্রমে $\hat{i}$ এর সাথে $\frac{\pi}{3}$ কোণ, $\hat{j}$ এর সাথে $\frac{\pi}{4}$ কোণ এবং $\hat{k}$ এর সাথে সমান সূক্ষ্মকোণ θ করে। θ এর মান নির্ণয় কর এবং $\hat{a}$ এর উপাংশগুলি নির্ণয় কর।

Sol^n We know,

$$\hat{a} = (\cos\alpha)\hat{i} + (\cos\beta)\hat{j} + (\cos\gamma)\hat{k} \longrightarrow (1)$$

Here, given,

$$\alpha = \frac{\pi}{3}, \beta = \frac{\pi}{4} \text{ & } \gamma = \theta.$$

$\therefore$ $\because \cos\alpha = \cos\frac{\pi}{3} = \frac{1}{2}$

$$\cos\beta = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$\therefore$ (1) $\Rightarrow \hat{a} = \frac{1}{2}i + \frac{1}{\sqrt{2}}\hat{j} + \cos\theta\,\hat{k} \longrightarrow (2)$

Again, $|\hat{a}| = 1$

$$\Rightarrow \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta} = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1^2 = 1$$

$$\Rightarrow \cos^2\theta = 1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{2}, \quad \because 0 \le \theta < \pi/2$$

$$\Rightarrow \theta = \pi/3 \leftarrow \text{Ans.}$$

$\therefore$ (2) $\Rightarrow \hat{a} = \frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{2}\hat{k}$

& components (উপাংশ) of $\hat{a}$ are: $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$ ← Ans. //