অনুশীলনী 10.3
Ex. 10.3

Q. No. (1) $7\hat{i}-\hat{j}+8\hat{k}$ ভেক্টরের $\hat{i}+3\hat{j}+7\hat{k}$ ভেক্টরের অভিক্ষেপ নির্ণয় কর।

find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

Solⁿ We know (আমরা জানি)

projection of $\vec{a}$ on $\vec{b}$ is $= \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

($\vec{b}$ ভেক্টরে $\vec{a}$ ভেক্টরের প্রক্ষেপ)

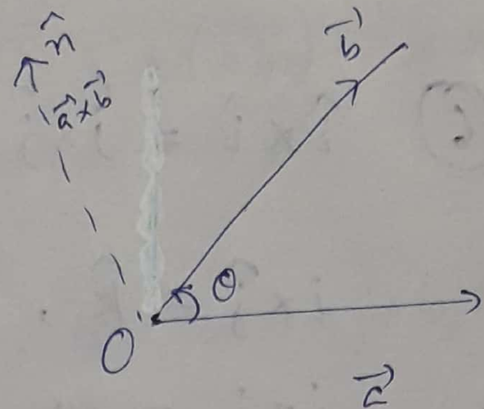
$\therefore$ projection of $\hat{i}+3\hat{j}+7k$ on the vector $7\hat{i}-\hat{j}+8k$ is

$$= \frac{(\hat{i}+3\hat{j}+7k)\cdot(7\hat{i}-\hat{j}+8\hat{k})}{|7\hat{i}-\hat{j}+8\hat{k}|}$$

$$= \frac{1\cdot 7+3\cdot(-1)+7\cdot 8}{\sqrt{7^2+(-1)^2+8^2}}$$

$$= \frac{60}{\sqrt{114}} \leftarrow \text{Ans}$$

দুটা ভেক্টৰৰ ভেক্টৰ (বা ক্ৰছ) পূৰণ:
(vector (or cross) product of two vectors)



Defn.

$$\boxed{\vec{a}\times\vec{b} = |\vec{a}|\,|\vec{b}|\sin\theta\ \hat{n}} \longrightarrow (1)$$

য'ত θ হ'ল $\vec{a}$ আৰু $\vec{b}$-ৰ মাজৰ কোণ, $0 \le \theta \le \pi$ আৰু $\hat{n}$ হ'ল এটা একক ভেক্টৰ যাতে $\vec{a}$ ৰ পৰা $\vec{b}$ লৈ ঘূৰোৱা সোঁহাতীয়া প্ৰণালী বা স্ক্ৰু $\hat{n}$-ৰ দিশত আগবাঢ়ে।

[where θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$ and $\hat{n}$ is a unit vector such that a right handed screw, when rotates from $\vec{a}$ to $\vec{b}$, advances in the direction of $\hat{n}$]

পৰ্যবেক্ষণ (Observation):

1. $\vec{a}\times\vec{b}$ এটা ভেক্টৰ (is a vector)
2. $\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$
3. $\vec{a}\times\vec{b} = \vec{0} \Leftrightarrow \vec{a} \parallel \vec{b}$
4. $\theta = \frac{\pi}{2} \Rightarrow |\vec{a}\times\vec{b}| = |\vec{a}|\,|\vec{b}|$

(5) $\sin\theta = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}||\vec{b}|}$

(6) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$, $\because$ here $\theta = \pi/2$
& $\sin \pi/2 = 0$.

$i\times\hat{j} = \hat{k}$, $j\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = j$

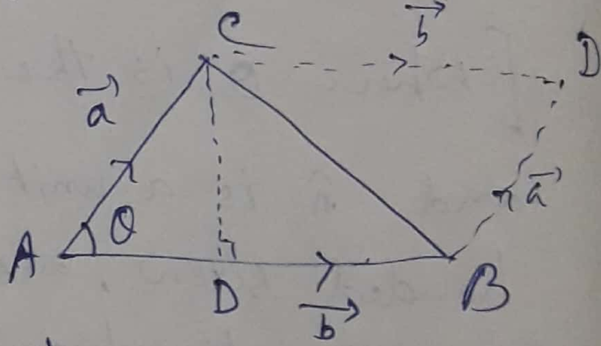
$j\times i = -k$, $k\times j = -i$, $\hat{i}\times\hat{k} = -j$

(7) ত্রিভুজের ক্ষেত্রফল (Area of triangle):

$\Delta ABC = \frac{1}{2} AB . CD$

$= \frac{1}{2}|\vec{b}||\vec{a}|\sin\theta$

$= \frac{1}{2}|\vec{a}\times\vec{b}|$



$\sin\theta = \dfrac{CD}{|\vec{a}|}$

$\Rightarrow CD = |\vec{a}|\sin\theta$

(8) $2\Delta ABC = |\vec{a}\times\vec{b}|$

$\Rightarrow$ Area of parallelogram ABDC $= |\vec{a}\times\vec{b}|$
(সামান্তরিক)

Note: i) $\frac{1}{2}|\vec{a}\times\vec{b}|$ is called scalar area of triangle ABC

ii) $\frac{1}{2}(\vec{a}\times\vec{b})$ " " vector " " " ABC.

Property (ধর্ম).

(i) $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$

(ii) $\lambda(\vec{a}\times\vec{b}) = (\lambda\vec{a})\times\vec{b} = \vec{a}\times(\lambda\vec{b})$, λ একটি স্কেলার

ই.

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$.

Then
$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Pf:
$$\vec{a}\times\vec{b} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})\times(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$$
$$= a_1\hat{i}\times(b_1 i + b_2 j + b_3 k) + a_2 j\times(b_1 i + b_2 j + b_3 k) + a_3\hat{k}(b_1 i + b_2 j + b_3 k)$$
$$= a_1b_1(i\times i) + a_1b_2(i\times j) + a_1b_3(i\times k)$$
$$+ a_2b_1(j\times i) + a_2b_2(j\times j) + a_2b_3(j\times k)$$
$$+ a_3b_1(k\times i) + a_3b_2(k\times j) + a_3b_3(k\times k)$$
$$= 0 + a_1b_2 k + a_1b_3(-j) + a_2b_1(-k) + 0$$
$$+ a_2b_3(\hat{i}) + a_3b_1(j) + a_3b_2(-i) + 0$$
$$= (a_2b_3 - a_3b_2)i - (a_1b_3 - a_3b_1)j + (a_1b_2 - a_2b_1)\hat{k}$$
$$= \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$