... can be repeated."

Possible Growth Patterns

Solow next arises the question whether there is always a capital accumulation path consistent with any rate of growth of the labour force. For this purpose he introduces a new variable $r = \frac{K}{L}$ (the ratio of capital to labour)

i.e.,
$$r = \frac{K}{L}$$

or $K = rL$

or $K = r L_0 e^{nt}$ [from eq^n (4)]

or $\frac{dK}{dt} = nrL_0e^{nt} + L_0e^{nt}\frac{dr}{dt}$ (differentiating w.r.t time.)

or $\frac{dK}{dt} = L_0e^{nt}\frac{dr}{dt} + nrL_0e^{nt}$

or $\dot{K} = L_0e^{nt}\left(\frac{dr}{dt} + nr\right) \longrightarrow (6)$

~~Substituting~~ Combining ⑥ and ⑤, we have

$$\left(\frac{dr}{dt} + nr\right) L_0 e^{nt} = sF(K, L_0 e^{nt}) \rightarrow ⑦$$

$$\text{or } \left(\frac{dr}{dt} + nr\right) L_0 e^{nt} = s L_0 e^{nt} F\left(\frac{K}{L_0 e^{nt}}, 1\right) \rightarrow ⑦$$

[diving the prod fn by $L_0 e^{nt}$ and multiplying by the same factor to other]

Now, dividing both side of ⑦ by $L_0 e^{nt}$,

$$\frac{dr}{dt} + nr = sF\left(\frac{K}{L_0 e^{nt}}, 1\right)$$

$$\text{or } \frac{dr}{dt} = sF\left(\frac{K}{L_0 e^{nt}}, 1\right) - nr$$

$$\text{or } \frac{dr}{dt} = sF(r, 1) - nr \rightarrow ⑧ \quad \left[\because \frac{K}{L_0 e^{nt}} = r\right]$$

$$\text{or } r' = sF(r, 1) - nr \rightarrow ⑧ \quad \left[\because \frac{dr}{dt} = r'\right]$$

The function $F(r, 1)$ in this equation is the total product curve as varying amounts of r ~~of capital~~ are employed with one unit of labour. In other words, it gives output per worker as a function of capital per worker. Thus, equation ⑧ states that " the rate of change of capital labour ratio is the difference of two terms, one representing the increment of capital [$sF(r,1)$] and other the increment of labour (nr)

Solow illustrates the possible growth pattern based on equation ⑧ diagrammatically as shown in fig. 1. In this figure the curve through the origin represents the f^n nr. The other curve passing through the origin and convex upward is the function $sF(r,1)$. The implication is that for output to be positive, inputs have to be positive and that there is diminishing marginal productivity of capital.

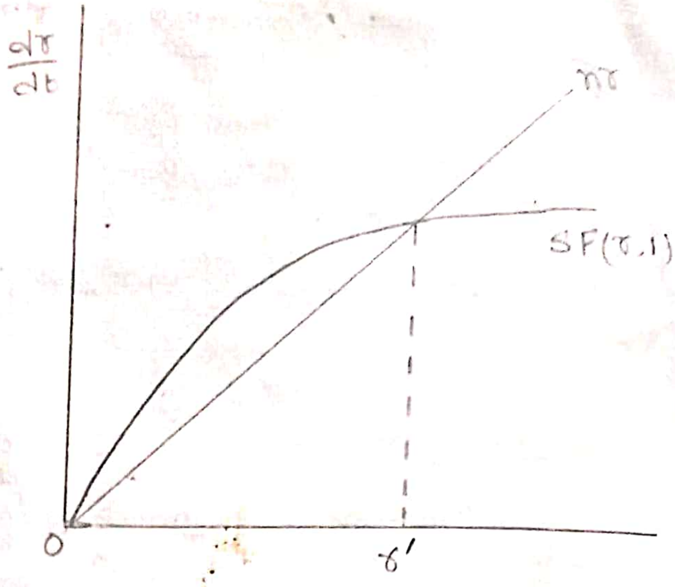

Fig. 1

At the point of intersection $nr = sF(r,1)$, which means that $\dot{r} = 0$ (from eq^n (8)). If the capital-labour ratio corresponding to this point r' is established, it will be maintained and, capital and labour will grow in proportion. Real output will also grow at the same relative rate as he assumes constant return to scale.

If there is any departure of r from r' then it automatically comes to r'. Say, If $r > r'$, we are towards the right of the intersection point. Here, $nr > sF(r,1)$, r will decrease towards r' (from eq^n (8)). On the other hand, if $r < r'$, $nr < sF(r,1)$, then $\dot{r} > 0$ and r will increase towards r'. This shows that on either side of r', there is a tendency to move towards r'. Therefore, eq^m value r' is stable. Thus, "whatever the initial value of capital-labour ratio, the system will develop towards a state of balanced growth at the natural rate... If the initial capital stock is below the eq^m ratio, capital & output will grow at a faster pace than the labour force until the eq^m ratio is approached. If the initial ratio is above the eq^m value, capital and output will grow more slowly than the labour force."

The basic conclusion that emerges from Solow's model is that "when prod^n takes place under the usual neo-classical cond^n of variable proportions and constant return to scale, no simple opposition bet^n natural and warranted rates of growth is possible. There may not be - in fact in the case of Cobb - Douglas function there can never be - any knife edge. ~~eq^m~~. The system can adjust to any given rate of growth of labour force, and eventually approach a state of