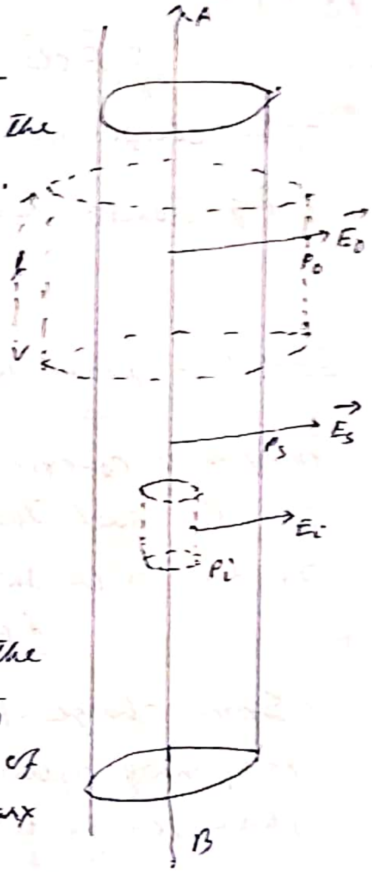
3. Electric field due to an infinitely long uniform cylindrical charge distribution

Consider an infinitely long cylinder of radius R and axis AB. Let ρ be the charge density i.e charge/unit volume. Due to the symmetry of the electric field the field strength is everywhere perpendicular to the curve surface while it is parallel to its flat surface. Therefore no electric flux is linked through the flat surface.



Case - I

Let P_0 be the point lying out of the cylinder at a distance r_0 from its axis. Imagine a co-axial cylinder of length l and radius r_0. Electric flux linked with this cylinder is

$\oint E_0 \, d\vec{s} = E_0 \, 2\pi r_0 l$. Where E_0 is the field strength at P_0.

~~at P_0~~. Total charge inside the cylinder

$$= \rho \times (\text{volume of the cylinder of radius } R \text{ and length } l)$$

$$= \rho \times \pi R^2 l.$$

According to Gauss Theorem

$$\oint \vec{E_i} \cdot d\vec{s} = \frac{1}{\epsilon_0 \epsilon_r} (\text{Total charge inside the cylinder})$$

$$\Rightarrow E_0 \, 2\pi r_0 l = \frac{1}{\epsilon_0 \epsilon_r} \times \pi R^2 l \rho.$$

$$E_0 = \frac{1}{2\pi \epsilon_0 \epsilon_r} \frac{\pi R^2 \rho}{r_0}$$

Let $\pi R^2 \rho = \mu$ (charge per unit length of the cylinder)

$$\therefore E_0 = \frac{1}{4\pi \epsilon_0 \epsilon_r} \frac{2\mu}{r_0} \quad \text{which is required value}$$

$$\therefore E_0 \propto \frac{1}{r}$$

Case II :- point lying on the surface of the cyllinder.

If E_s be the electric field on the surface of the cyllinder

$$\oint_s \vec{E_s}\cdot\vec{ds} = E_s \times 2\pi R l .$$

Total charge inside the cyllinder $= \rho \times \pi R^2 l$.

Using Gauss's law $E_s \times 2\pi R l = \frac{1}{\epsilon_0 \epsilon_r} \times \rho \times \pi R^2 l$.

$$E_s = \frac{1}{4\pi\epsilon_0\epsilon_r} \times 2\pi R \rho$$

Case III :- point lying inside the cyllinder :—

Imagine a co-axial cyllinder of length l and radius r_i ($r_i < R$) such that the point P_i situated at distance r_i lies on its surface. In this case

$$\oint \vec{E_i}\cdot\vec{ds} = E_i \times 2\pi r_i l .$$

Some charge of the charge distribution lies out of this imaginary cyllinder. It will not produce any electric flux through the cyllinder.

$$E_i \times 2\pi r_i l = \frac{1}{\epsilon_0\epsilon_r} \rho \pi r_i^2 l .$$

$$E_i = \frac{1}{4\pi\epsilon_0\epsilon_r} 2\pi r_i \rho .$$

$$\text{or } E_i = \frac{1}{2\epsilon_0\epsilon_r} r_i \rho .$$

$\therefore E_i \propto r_i$

Graphically variation of electric field due to a cyllindrical charge distribution with the distance from the axis is shown.