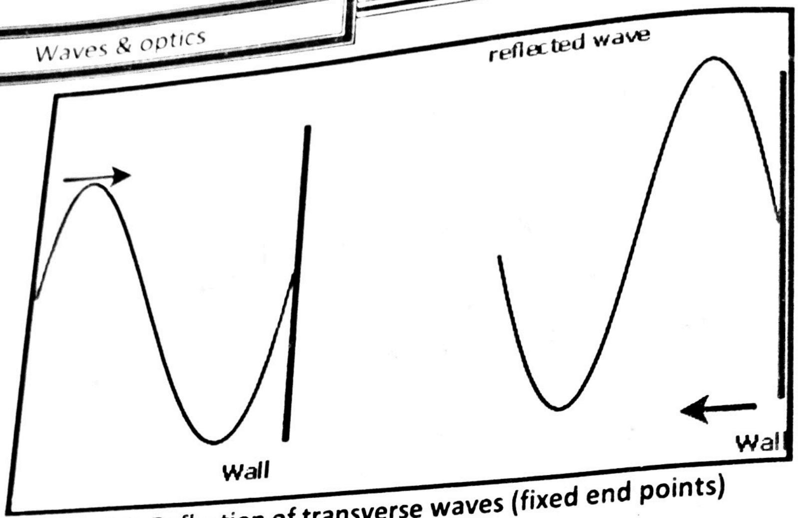

Fig. 2.3: Reflection of transverse waves (fixed end points)

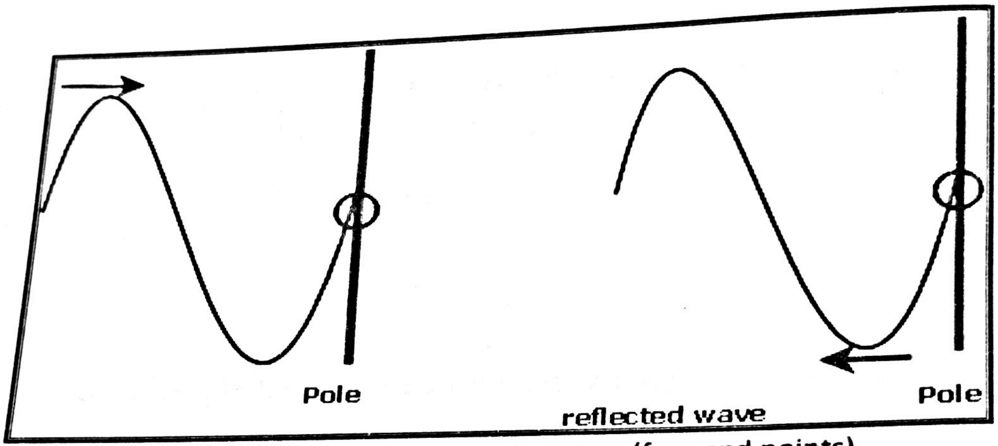

Fig. 2.4: Reflection of transverse waves (free end points)

2.5 Standing waves or stationary waves

When either of the two scenarios of wave reflection happens, the incident wave meets the reflected wave. These waves move past each other in opposite directions, causing interference. When these two waves have the same frequency, the product of this is called the ***standing waves***. Standing waves appear to be standing still, hence the name. To understand how standing waves occur, we can analyze them as follows

When the incident wave and reflected wave first meet, both waves have an amplitude is zero. As the waves continue to move past each other, they continue to interfere with each other either constructively of destructively.

Every point in the medium having a standing wave oscillates up and down and the amplitude of the oscillations depends on the location of the point. When we observe standing waves on strings, it looks like the wave is not moving and standing still. The principle of standing waves is the basis of resonance and how many musical instruments get their sound. The points in a standing wave that appear to remain flat and do not move are called ***nodes***. The points which reach the maximum oscillation height are called ***antinodes***.

When two sets of progressive wave trains of the same type (i.e, both longitudinal or both transverse) having the same amplitude and same time period/frequency/wavelength travelling with same speed along the same straight line in opposite directions superimpose, a new set of waves are formed. These are called stationary waves or standing waves.

There are two types of standing waves:

1) Longitudinal Stationary waves: are formed as a result of superimposition of two identical longitudinal waves travelling in opposite directions.

2) Transverse Stationary waves: are formed as a result of superimposition of two identical transverse waves travelling in opposite directions.

Characteristics of stationary waves

- The disturbance is confined to a particular region between the starting point and the reflecting point of the wave.
- There is no onward motion of the disturbance from one particle to the adjoining particle and so on beyond this particular region.
- The total energy correlated with a stationary wave is twice the energy of each of incident and reflected wave. But there is no flow or transference of energy along the stationary wave.

There are certain points in the medium in a standing wave, which are permanently at rest. These are called nodes. The distance between two consecutive nodes is $\frac{\lambda}{2}$.

- In a standing wave, the medium divides into a number of segments. Each segment is vibrating up and down as a whole.
- All the particles except those at nodes, execute simple harmonic motion about their mean position in the same time period.
- The amplitude of vibration of particles varies from zero at nodes to maximum at antinodes.
- Twice during each vibration, all the particles of the medium pass at the same time through their mean position.
- The wavelength and time period of stationary waves are the same as for the component waves.
- While crossing mean position, velocity of particles varies from maximum at antinodes to zero at nodes.

2.5.1 Standing waves – Fixed and Free Ends

Standing waves can form under a variety of conditions, but they are easily demonstrated in a medium which is finite or bounded. In general, standing waves can be produced by any two identical waves traveling in opposite directions that have the right wavelength. In a bounded medium, standing waves occur when a wave with the correct wavelength meets its reflection. The interference of these two waves produces a resultant wave that does not appear to move. When the *driving frequency* applied to a system equals its *natural frequency*. This condition is known as *resonance*. Standing waves are always associated with resonance. Resonance can be described by a dramatic increase in amplitude of the resultant vibrations. Compared to traveling waves with the same amplitude, producing standing waves is relatively effortless

Any system in which standing waves can form has several natural frequencies. The set of all possible standing waves are known as the *harmonics* of a system. The simplest of the harmonics is called the *fundamental* or first harmonic. Subsequent standing waves are called the second harmonic, third harmonic, etc. The harmonics above the fundamental, especially in music theory, are sometimes also called *overtones*. There are three simple cases for the wavelengths that will form standing waves in a simple, one-dimensional system.

One dimension: two fixed ends

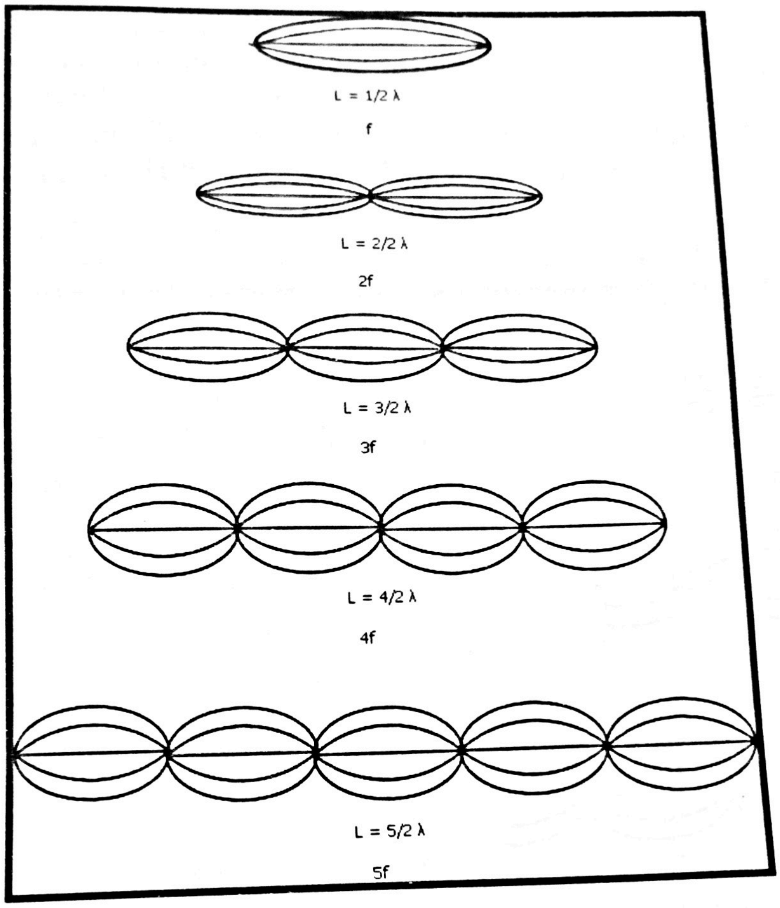

Fig 2. 5

If a medium is bounded such that its opposite ends can be considered fixed, nodes will then be found at the ends. The simplest standing wave that can form under these circumstances has one antinode in the middle. This is half a wavelength. To make the next possible standing wave, place a node in the center. We now have one whole wavelength. To make the third possible standing wave, divide the length int thirds by adding another node. This gives us one and a half wavelengths. It shoul become obvious that to continue all that is needed is to keep adding nodes, dividi the medium into fourths, then fifths, sixths, etc. There are an infinite number harmonics for this system, but no matter how many times we divide the medium we always get a whole number of half wavelengths $\left(\frac{1}{2}\lambda . \frac{2}{2}\lambda, \frac{3}{2}\lambda, \ldots, \frac{n}{2}\lambda\right)$

There are important relations among the harmonics themselves in this sequence. The wavelengths of the harmonics are simple fractions of the fundamental wavelength.

the fundamental wavelength were 1 m the wavelength of the second harmonic would be $\frac{1}{2}$ m, the third harmonic would be $\frac{1}{3}$ m, the fourth $\frac{1}{4}$ m, and so on. Since frequency is inversely proportional to wavelength, the frequencies are also related. The frequencies of the harmonics are whole-number multiples of the fundamental frequency. If the fundamental frequency were 1 Hz the frequency of the second harmonic would be 2 Hz, the third harmonic would be 3 Hz, the fourth 4 Hz, and so on.

One dimension: two free ends

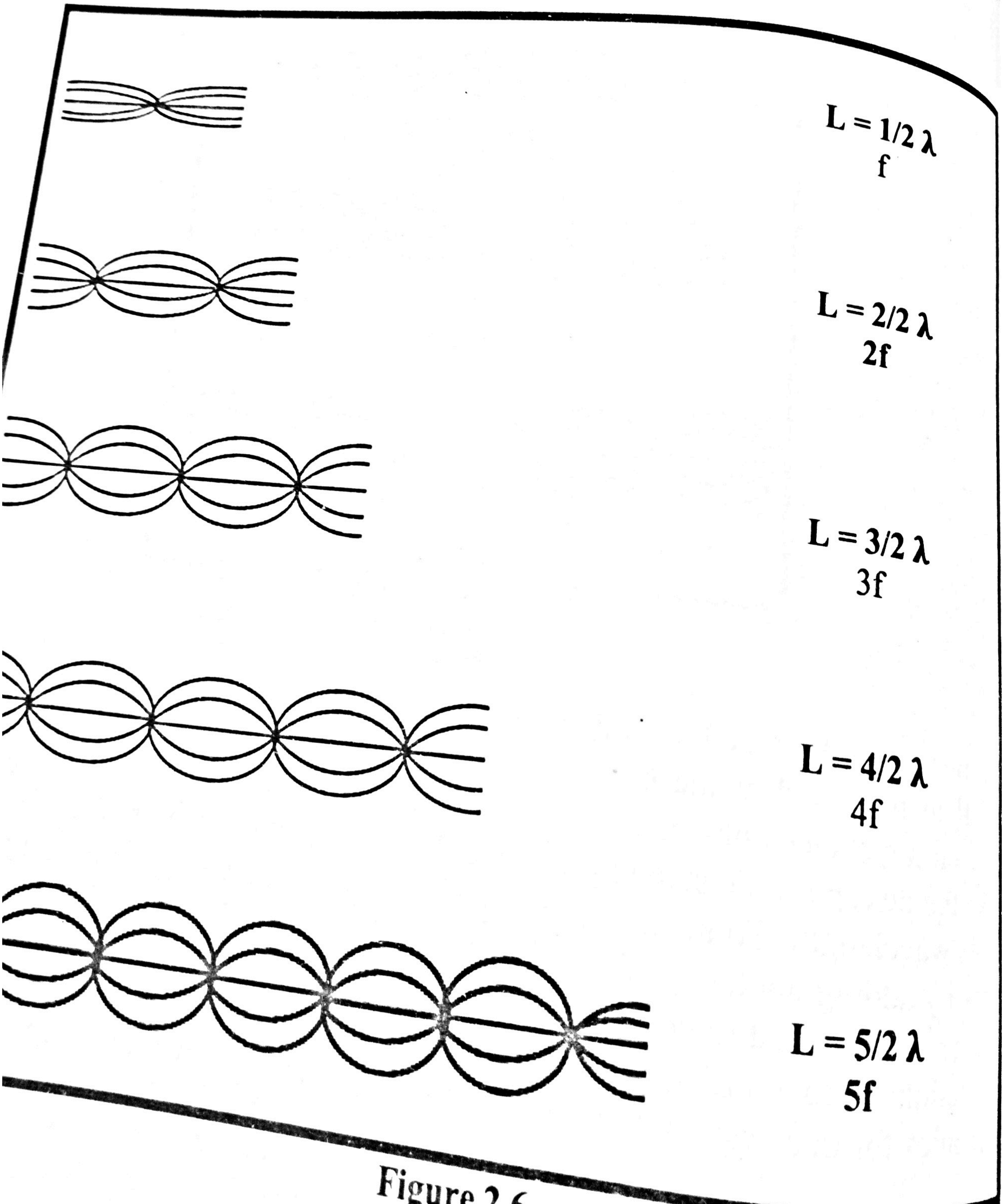

Figure 2.6

If a medium is bounded such that its opposite ends can be considered free, antinodes will then be found at the ends. The simplest standing wave that can form under these circumstances has one node in the middle. This is half a wavelength. To make the next possible standing wave, place another antinode in the center. We now have one whole wavelength. To make the third possible standing wave, divide the length into thirds by adding another antinode. This gives us one and a half wavelengths. It should become obvious that we will get the same relationships for the standing waves formed between two free ends that we have for two fixed ends. The only difference is that the nodes have been replaced with antinodes and vice versa. Thus when standing waves form in a linear medium that has two free ends a whole number of half wavelengths fit inside the medium and the overtones are whole number multiples of the fundamental frequency

One dimension: one fixed end — one free end

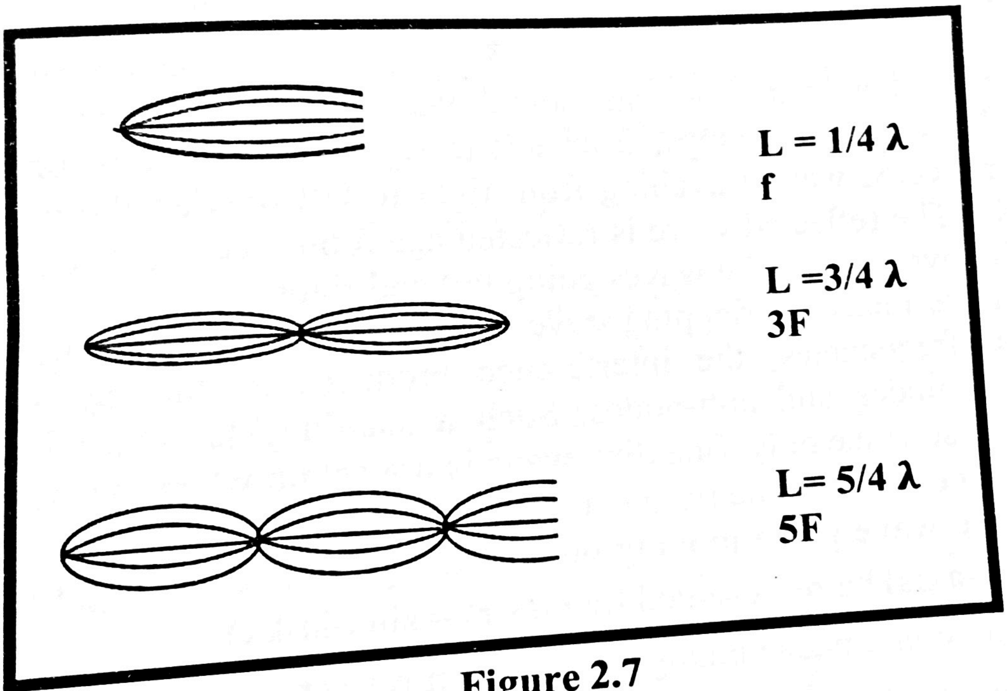

Figure 2.7

When the medium has one fixed end and one free end the situation changes in an interesting way. A node will always form at the fixed end while an antinode will always form at the free end. The simplest standing wave that can form under these circumstances is one-quarter wavelength long. To make the next possible standing waves add both a node and an antinode, dividing the drawing up into thirds. We now have three-quarters of a wavelength. Repeating this procedure we get five-quarters of a wavelength, then seven-quarters, etc. In this arrangement, there are always an odd number of quarter wavelengths present.

Thus the wavelengths of the harmonics are always fractional multiples of the fundamental wavelength with an odd number in the denominator. Likewise frequencies of the harmonics are always odd multiples of the fundamental frequency.

The three cases above show that, although not all frequencies will result in standing waves, a simple, one-dimensional system possesses an infinite number of natural frequencies that will. It also shows that these frequencies are simple multiples of some fundamental frequency. For any real-world system, however, the higher frequency standing waves are difficult if not impossible to produce. Tuning forks, for example, vibrate strongly at the fundamental frequency, very little at the second harmonic, and effectively not at all at the higher harmonics.

2.5.2 Standing waves in strings and normal modes of vibrations

When we send a continuous sinusoidal wave of certain frequency along a string stretched between two clamps, from left to right, it is reflected on reaching the right end. The reflected wave travelling from right to left overlap the wave travelling from left to right. The reflected wave is reflected again on reaching the left and travels from left to right, overlapping the waves going left and right.

Soon, we have many overlapping travelling waves, which interfere with one another. For certain frequencies, the interference produces a standing wave pattern with formation of nodes and anti-nodes. Such a standing/stationary wave is said to be produces at resonance only. Standing wave is not set up when the string is oscillating at any frequency other than the resonant frequency.

Let the wave pulse moving on the string from left to right (i.e. along positive direction of x-axis) be represented by $y_1(x,r) = \sin(\omega t\text{-}kx)$

As there is a phase change of π radian reflection at the fixed end of the string therefore, the reflected wave pulse travelling from right to left on the string represented by

$$y_2(x,t) = r\sin(\omega t + kx + \pi) = -r\sin(\omega t + kx)$$

According to superposition principle, the resultant displacement y at time t at position x is given by

$$y(x,y) = y_1(x,t) + y_2(x,t)$$

$y(x,t) = r\sin(\omega t - kx) - r\sin(\omega t + kx) = -r[\sin(\omega t + kx) - \sin(\omega t - kx)]$ ... (1)

Using the relation: $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$, we get

$y(x,t) = -2r\cos\omega t\sin kx$ ... (2)

$y\,(x,y) = -(-2\,r\sin kx)\cos\omega t$ ... (3)

Equation (3) do not represent a moving harmonic wave i.e., the disturbance or the waveform does not move to either side. The quantity within the bracket, i.e. (2 r sin kx) is the amplitude of oscillation of the element of the string located at position x. For different value of x, the amplitude is different. This is in contrast to a progressive wave, where amplitude of wave, where amplitude of wave is same for all elements. Eqn. (3) therefore, represents a standing wave, i.e., a wave in which the wave form does not move.

\t one end of the string, where x = 0, from (3),

$$y = -2\,r\cos\omega t\,\sin 0^0 = 0$$

'his end is a node.

the other end of the string, where x = L

n(3),

-2 r cosωt sin k L ...(4)

e other end of the string is fixed,

y = 0, at this end. This end is also a node.

4), for y = 0 sin k L = 0 = sin n π, where, n = 0,1,2,3....

$= n\pi$ or $\frac{2\pi}{\lambda}L = n\pi$ or $\lambda = \frac{2L}{n}$ ... (5)

t n = 1, 2, 3... Correspond to 1st, 2nd, 3rd.... normal modes of vibratic

(i) First normal mode of vibration

Suppose λ_1 is the wavelength of standing waves set up on the string correspo[nding] to n = 1.

From (5), $$\lambda_1 = \frac{2L}{1} \text{ or } L = \frac{\lambda_1}{n}$$

The string vibrates as a whole in the one segment, as shown in Fig. 2.8 a. The[re are] two nodes at the two fixed ends and one antinode in the middle as shown.

The frequency of vibration is given by

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \quad \text{... (6)}$$

As, $$v = \sqrt{T/m}$$

Where T is tension in the string and m is mass per unit length of the string.

∴ from (6), $$\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} \quad \text{... (7)}$$

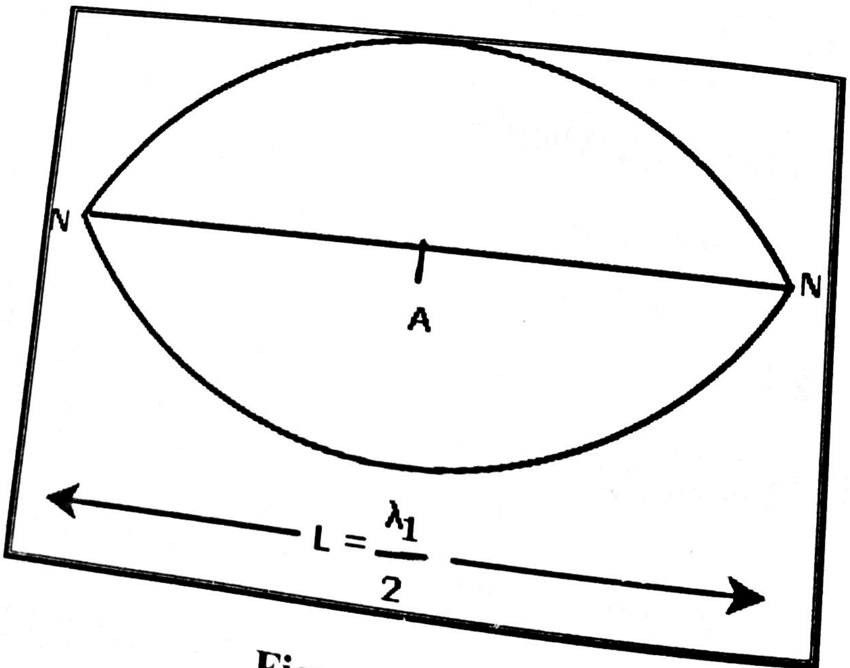

Figure 2.8(a)

This (first) normal made of vibration is called fundamental mode. The frequenc[y of] vibration (ν_1) of string in this made is minimum and is called fundamental freque[ncy.] The sound or note so produced s called fundamental note or first harmonic.

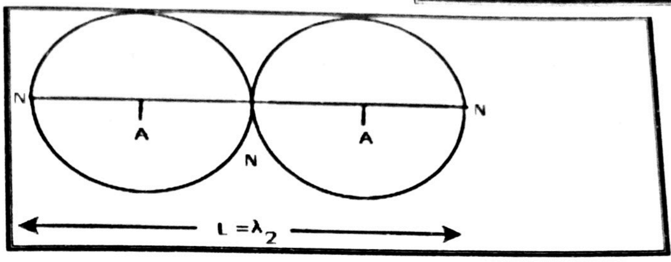

Figure 2.8(b)

(ii) Second normal mode of vibration

Let λ_2 be the wavelength of standing waves set up on the string corresponding to n = 2.

From (5), $\lambda_2 = \dfrac{2L}{2} = L$

The string vibrates in two segments of equal length, as shown in Fig.2.8 (b). There are three nodes and two antinodes as shown. The frequency of vibration is given by

$$v_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2 \times \frac{v}{2L}$$

$$v_2 = 2\,v_1 \qquad \text{... (8)}$$

i.e., frequency of vibration of string becomes twice the fundamental frequency. The note so produced is called second harmonic or first overtone.

(iii) Third normal mode of vibration

Let λ_3 be the wavelength of standing waves set up on the string corresponding to n = 3.

From (5)$\lambda_3 = \dfrac{2L}{3}$ or $L = \dfrac{3\lambda_3}{2}$

The string vibrates in three segments of equal length, as shown in Fig.2.8(b) 2.8(
There are four nodes and three antinodes as shown.

The frequency of vibration is given by $v_3 = \dfrac{v}{\lambda_3} = \dfrac{v}{\frac{2L}{3}} = 3\left(\dfrac{v}{2L}\right) v_3 = 3v_1$ ... (9 i.e., frequency of vibration of string becom... The note so produced is called third harmonic or second overtone.

In general, the wavelength of nth mode of vibration of string is, λ_n

$$= \frac{2L}{n} \quad [from(5)]$$

The corresponding frequency of vibration would be $\nu_n = \frac{v}{\lambda_n} = \frac{v}{2L/n} = n\left(\frac{v}{2L}\right)$ ... (10)

$$V_n = nV_1$$

This frequency is n times the fundamental frequency. The note so produced ... harmonic or $(n-1)^{th}$ overtone.

Note: The collection of all possible oscillation modes is called the harmonic serie... n is called the harmonic number of the n^{th} harmonic.

Position of Nodes:

In standing waves, we know that nodes are the positions of zero displacem... They are represented by n. From Fig. 2.8(b), we observe that there are two node... the first normal made of vibration, three nodes in the second normal mode, ... nodes in the 3rd normal more of vibration and so. Therefore, in the nth nor... made vibration, there will be (n+1) nodes. These nodes are located at the distanc...

$$x = 0, \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n} \ldots . L.$$

For example, in 1st normal mode, n = 1.

The nodes are at $x = 0, x = \frac{L}{1}, = L$, Fig. 2(b)1(a)

In 2nd normal mode of vibration, n = 2

The nodes are at $x = 0, x = \frac{L}{2}, x = \frac{2L}{2}L$

These are shown in Fig. 2.8(b), and so on.

Position of Antinodes:

In standing waves, antinodes are the position of maximum displacement. They are represented by A. As is clear from Fig. 2.8 (b), there will be n antinodes in the nth normal mode of vibration.

Antinode are located in between nodes, therefore, their position will be given by

$$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \ldots, \frac{(2n-1)L}{2n}$$

For example, in first normal mode, n = 1, the antinode is at x $\frac{L}{2\times 1} = \frac{L}{2}$, Fig. 2(b)1(a)

In 2nd normal mode of vibration, n = 2, the antinodes are at $x = \frac{L}{2\times 2} = \frac{L}{4}$ and $x = \frac{3L}{2\times 2} = \frac{3L}{4}$

These are shown in Fig 2.8 (b), and so on.

2.6 Longitudinal Waves

A longitudinal wave is the one that moves parallel to the direction of waves of particles in motion. That is a straight parallel line above the particle. For instance in the same rope kept horizontally, if one introduces a pulse on the left and the right end, the energy flows from both ends trapping the movements in a parallel motion. These are longitudinal waves.

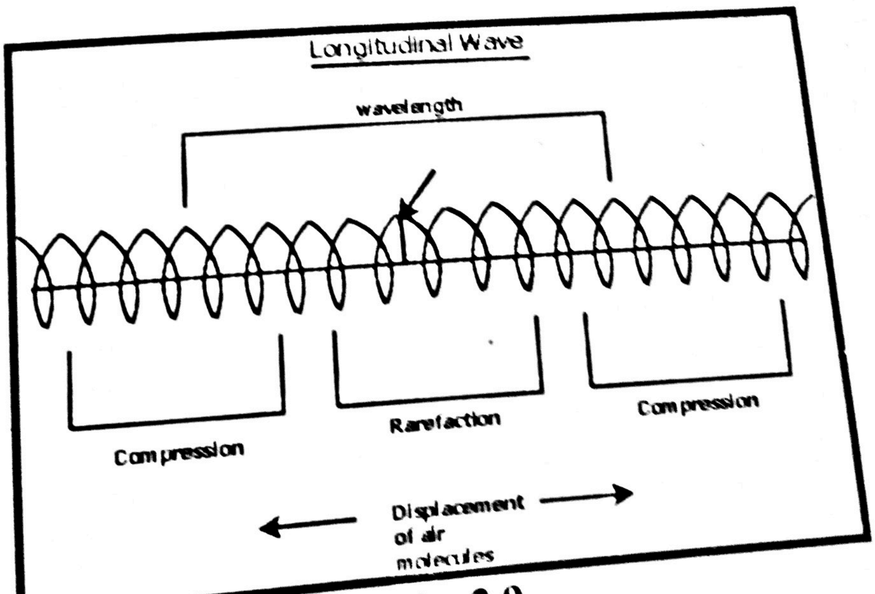

Fig. 2.9