অনুশীলনী 10.3

Q. No. 16. দেখুওৱা যে $A(1,2,7)$, $B(2,6,3)$ and $C(3,10,-1)$ বিন্দু কেইটা একৰেখীয় (colinear).

Sol: We have (আমাৰ পাওঁ)

A বিন্দুৰ অৱস্থান ভেক্টৰ $= \hat{i}+2\hat{j}+7k$

(p.v. of the point A)

B " " " $= 2i+6j+3k$

C " " " $= 3i+10j-k$

Now,

$$\overrightarrow{AB} = (\text{p.v. of } B) - (\text{p.v. of } A)$$

$$= (2i+6j+3k)-(i+2j+7k) = i+4j-4k$$

$$\overrightarrow{BC} = (\text{p.v. of } C) - (\text{p.v. of } B)$$

$$= (3i+10j-k)-(2i+6j+3k) = i+4j-4k$$

$$\overrightarrow{AC} = (3i+10j-k)-(i+2j+7k) = 2i+8j-8k$$

$$\therefore |\overrightarrow{AB}| = \sqrt{1^2+4^2+(-4)^2} = \sqrt{33}$$

$$|\overrightarrow{BC}| = \sqrt{1^2+4^2+(-4)^2} = \sqrt{33}$$

$$|\overrightarrow{AC}| = \sqrt{2^2+8^2+(-8)^2} = \sqrt{4(1+16+16)} = 2\sqrt{33}$$

$$\therefore |\overrightarrow{AB}| + |\overrightarrow{BC}| = |\overrightarrow{AC}|.$$

Hence the points A, B, C are co-linear.

(গতিকে A, B, C বিন্দু তিনটা একৰেখীয়)

Exercise 10.3.

Ex. 17. দেখাও যে $2i-j+k$, $i-3j-5k$ এবং $3i-4j-4k$ ভেক্টর তিনটি একটি সমকোণী ত্রিভুজের শীর্ষবিন্দু গঠন করে।

সমা: ধরি, বিন্দু তিনটি A, B, C. অর্থাৎ $A(2i-j+k)$, $B(i-3j-5k)$ এবং $C(3i-4j-4k)$.

Now,

$$\overrightarrow{AB} = (i-3j-5k)-(2i-j+k) = -i-2j-6k$$

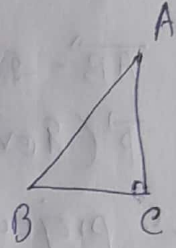
$$\overrightarrow{BC} = (3i-4j-4k)-(i-3j-5k) = 2i-j+k$$

$$\overrightarrow{AC} = (3i-4j-4k)-(2i-j+k) = i-3j-5k$$

$$\therefore |\overrightarrow{AB}| = \sqrt{(-1)^2+(-2)^2+(-6)^2} = \sqrt{41}$$

$$|\overrightarrow{BC}| = \sqrt{2^2+(1)^2+1^2} = \sqrt{6}$$

$$|\overrightarrow{AC}| = \sqrt{1^2+(-3)^2+(-5)^2} = \sqrt{35}$$



Thus, $|\overrightarrow{AB}| < |\overrightarrow{BC}| + |\overrightarrow{AC}|$ [In fact, $|\overrightarrow{AB}| = |\overrightarrow{BC}| + |\overrightarrow{AC}|$]

$\therefore$ A, B, C একরেখীয় নহে (A, B, C are not co-linear).

$\Rightarrow$ A, B, C একটি ত্রিভুজের শীর্ষবিন্দু (A, B, C are virtices of a triangle).

এখন (Now),

$$(\overrightarrow{BC})\cdot(\overrightarrow{AC}) = (2i-j+k)\cdot(i-3j-5k)$$
$$= 2.1+(-1).(-3)+1.(-5)$$
$$= 2+3-5 = 0$$

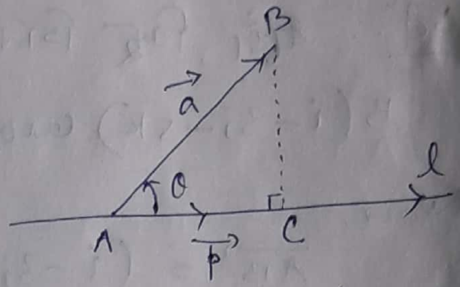
$$\Rightarrow \overrightarrow{BC} \perp \overrightarrow{AC}.$$

$\therefore$ A, B, C একটি সমকোণী ত্রিভুজের শীর্ষবিন্দু, (A, B, C are virtices of a right angle triangle.) //

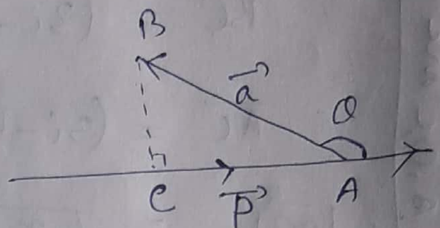
Note: We see $|\overrightarrow{AB}|^2 = 41$, $|\overrightarrow{BC}|^2 + |\overrightarrow{AC}|^2 = 6+35 = 41$.
Hence $|\overrightarrow{AB}|^2 = |\overrightarrow{BC}|^2 + |\overrightarrow{AC}|^2$
$\Rightarrow$ By Pythagoras theorem, $\therefore$ A, B, C are virtices of a right angle triangle.

ৰেখাত ভেক্টৰৰ প্ৰক্ষেপ
(Projection of a vector on a line)

ধৰা হ'ল $\overrightarrow{AB}$ ভেক্টৰে এডাল নিৰ্দিষ্ট ৰেখা l-ৰ লগত ঘড়ীৰ কাঁটাৰ বিপৰীত দিশত θ কোণ কৰে।
(Let $\overrightarrow{AB}$ makes an ange θ will the line l in anticlockwise direction).



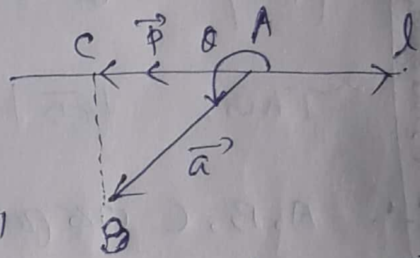
$0^\circ < \theta < 90^\circ$ fig (i)



$(90^\circ < \theta < 180^\circ)$ fig (ii)

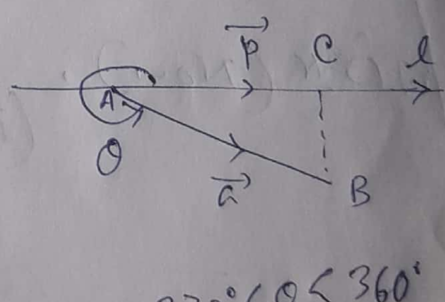
Then, ভেক্টৰ l-ৰ ওপৰত $\overrightarrow{AB}$-ৰ অভিক্ষেপ হ'ল এটা ভেক্টৰ $\vec{p}$ (ধৰা হ'ল), য'ত $|\vec{p}| = |\overrightarrow{AB}|\cos\theta$.
i.e. projection of $\overrightarrow{AB}$ on l is $= \vec{p}$,
where, $|\vec{p}| = |\overrightarrow{AB}|\cos\theta$.

$\vec{p}$-ক অভিক্ষেপ ভেক্টৰ (projection vector) কোৱা হয় আৰু $|\vec{p}|$-ক অভিক্ষেপ (projection) বোলে।



$180^\circ < \theta < 270^\circ$ fig (iii)

Note:
$$|\vec{p}| = |\overrightarrow{AB}|\cos\theta = \frac{|\overrightarrow{AB}||\vec{l}|\cos\theta}{|\vec{l}|} = \frac{\overrightarrow{AB}\cdot\vec{l}}{|\vec{l}|} = \overrightarrow{AB}\cdot\frac{\vec{l}}{|\vec{l}|} = \overrightarrow{AB}\cdot\hat{l}$$



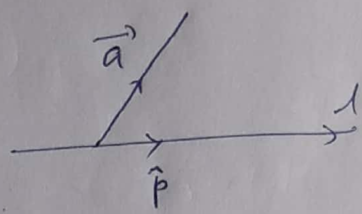
$270^\circ < \theta < 360^\circ$ fig (iv)

$\Rightarrow$ Projection (অভিক্ষেপ) $= \overrightarrow{AB}\cdot\hat{l}$, য'ত $\vec{l}$ ৰ দিশত একক ভেক্টৰ $\hat{l}$.

পৰ্যবেক্ষণ :
(Observation)

① যদি এডাল ৰেখা l ত $\hat{p}$ এটা একক ভেক্টৰ হয়, তেন্তে l ৰেখাডালত $\vec{a}$ ভেক্টৰৰ অভিক্ষেপ $= \vec{a}\cdot\hat{p}$

i.e. projection of $\vec{a}$ on l is $= \vec{a}\cdot\hat{p}$,



(where $\hat{p}$ is the unit vector on the line l.

(**) ② $\vec{b}$ ভেক্টৰত আন এটা ভেক্টৰ $\vec{a}$-ৰ অভিক্ষেপ $= \vec{a}\cdot\hat{b} = \vec{a}\cdot\left(\frac{\vec{b}}{|\vec{b}|}\right)$
(projection of $\vec{a}$ on $\vec{b}$)

$$= \frac{1}{|\vec{b}|}(\vec{a}\cdot\vec{b})$$

সেইদৰে, $\vec{a}$-ত $\vec{b}$-ৰ অভিক্ষেপ $= \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$
(projection of $\vec{b}$ on $\vec{a}$)

অনুশীলনী 10.3

Q.No.(3). $\hat{i}+\hat{j}$ ভেক্টৰত $\hat{i}-\hat{j}$ ভেক্টৰৰ অভিক্ষেপ নিৰ্ণয় কৰা।

Sol: We know, projection of $\vec{a}$ on $\vec{b}$ is $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$, → $\boxed{\text{By observation No.(2)}}$

∴ projection of $\hat{i}-\hat{j}$ on $\hat{i}+\hat{j}$ is $= \frac{(\hat{i}-\hat{j})\cdot(\hat{i}+\hat{j})}{|\hat{i}+\hat{j}|}$
($\hat{i}+\hat{j}$ ভেক্টৰত $\hat{i}-\hat{j}$ ভেক্টৰৰ অভিক্ষেপ হ'ব)

$$= \frac{\hat{i}\cdot\hat{i}+\cancel{\hat{i}\cdot\hat{j}}-\cancel{\hat{j}\cdot\hat{i}}-\hat{j}\cdot\hat{j}}{\sqrt{1^2+1^2}}$$

$$= \frac{1-1}{\sqrt{2}} = 0 \leftarrow \text{Ans.}$$