Velocity: The rate of change of displacement with time is called velocity. So if $x = x(t)$ be the position of a particle at any time t, then velocity is

$$v = \frac{dx(t)}{dt} \longrightarrow (*)$$

Here we consider the magnitude of displacement and velocity

Acc^n: The rate of change of velocity with time is called acc^n. So acc^n is

$$a(t) = \frac{dv}{dt} \longrightarrow (1)$$

$$= \frac{d}{dt}\left(\frac{dx(t)}{dt}\right)$$

$$= \frac{d^2x(t)}{dt^2}$$

Also, $a(t) = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{dv}{dx}\cdot v$

By Newton's 2nd law,

$$F = ma \quad , \text{ where } F \text{ is force, } m \text{ is the mass of the particle and } a \text{ is acc}^n.$$

$$= m\frac{dv}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \frac{F}{m} = a$$

If F is constant, m is const, then $\frac{F}{m} = a$ (acc^n) is also constant.

$\therefore (1) \Rightarrow$

$$\frac{dv}{dt} = a \quad , \longrightarrow (2) \quad \text{(here 'a' is constant and not function of 't')}$$

Integrating,

$$\int \frac{dv}{dt}dt = \int a\,dt + c_1 \text{ , } c_1 \text{ is const. of integration}$$

$$\Rightarrow \quad v = at + c_1 \text{ , } \longrightarrow (3)$$

Let $v = v_0$ at $t = 0$. Then (3) gives $c_1 = v_0$.

$$\therefore (3) \Rightarrow \boxed{v = at + v_0} \text{ , } \longrightarrow (4)$$

This is the velocity function.

Note: Actually, (*), (1), (2) are diff. Eqn. Solving these eqns., we get diff. results, i.e. velocity function, distance function etc. at any time.

Now, we put $v = \frac{dx}{dt}$ in (4) and, get

$$\frac{dx}{dt} = at + v_0$$

or, $dx = (at + v_0)\,dt$

. Integrating,

$$\int dx = \int (at + v_0)\,dt + c_2$$

$$\Rightarrow x = \frac{1}{2}at^2 + v_0 t + c_2 \longrightarrow (5)$$

Let $x = 0$ at $t = 0$. Then above eqn. gives

$$0 = 0 + 0 + c_2$$

$$\Rightarrow c_2 = 0.$$

Putting this value of c_2 in (5), we get

$$\boxed{x = \frac{1}{2}at^2 + v_0 t} \longrightarrow (6)$$

which gives the position of the particle at any time t.

Again, we have

$$a = \frac{dv}{dt}$$

$$= \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} \cdot v$$

$$\Rightarrow a\,dx = v\,dv$$

$$\Rightarrow \int a\,dx = \int v\,dv + c_3 \text{ ; integrating}$$

$$\Rightarrow ax = \frac{v^2}{2} + c_3 \longrightarrow (7)$$

Let $v = v_0$, when $x = 0$. So above eqn. gives

$$a \cdot 0 = \frac{v_0^2}{2} + c_3$$

$$\Rightarrow c_3 = -\frac{v_0^2}{2}.$$

Putting this value of c_3 in (7), we get

$$ax = \frac{v^2}{2} - \frac{v_0^2}{2}$$

$$\Rightarrow \boxed{v^2 = v_0^2 + 2ax} \longrightarrow (8)$$

which gives the velocity of the particle at any dist. x.

Note: [Eqns (4), (6) and (8) are called the eqns. of motion of a particle of constant mass in a straight line under a constant force and constant accel^n.

OR → Eqns. of rectilinear motion under constant accel^n.

Ex. Find the vel. function $v(t)$ and position function $x(t)$ of a moving particle with the given accl^n. $a(t)$, initial position $x_0 = x(t)$, and initial velocity $v_0 = v(0)$ where $a(t) = 50$, $V_0 = 10$, $x_0 = 20$.

Sol^n: We know that

$$a(t) = \frac{dv}{dt} \longrightarrow (1)$$

Putting $a(t) = 50$ in (1), we get

$$\frac{dv}{dt} = 50$$

$$\Rightarrow dv = 50\, dt$$

$$\Rightarrow \int dv = \int 50\, dt + C_1$$

$$\Rightarrow v = 50t + C_1 \longrightarrow (2)$$

Initially, $v = v_0 = 10$, when $t = 0$. So putting these in (2)

$$10 = 50.0 + C_1 \Rightarrow C_1 = 10$$

Putting $C_1 = 10$ in (2), we get

$$v = 50t + 10 \longrightarrow (3)$$

which is the velocity function.

Again,

$$v = \frac{dx}{dt}$$

$$\Rightarrow 50t + 10 = \frac{dx}{dt}, \text{ by } (3)$$

$$\Rightarrow dx = (50t + 10)\, dt$$

$$\Rightarrow \int dx = \int (50t + 10)\, dt + C_2$$

$$\Rightarrow x = 25t^2 + 10t + C_2 \longrightarrow (4)$$

Initially, $x = x_0 = 20$ when $t = 0$. So (4) gives

$$20 = 25.0 + 10.0 + C_2 \Rightarrow C_2 = 20$$

$\therefore$ (4) $\Rightarrow$ $x = 25t^2 + 10t + 20$

which is the required position function.