Ex-3 If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \\ 6 & 3 \end{bmatrix}_{3\times 2}$ $B = \begin{bmatrix} 5 & 2 \\ 7 & 3 \\ 1 & 0 \end{bmatrix}_{3\times 2}$ and $C = \begin{bmatrix} 6 & 0 \\ -4 & 3 \\ 2 & 3 \end{bmatrix}_{3\times 2}$

Then prove that $(A+B) - C = A + (B-C)$

Sol^n:-

$$A + B = \begin{bmatrix} 2 & 3 \\ 5 & 7 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 7 & 3 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2+5 & 3+2 \\ 5+7 & 7+3 \\ 6+1 & 3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 5 \\ 12 & 10 \\ 7 & 3 \end{bmatrix}$$

$$\therefore (A+B) - C = \begin{bmatrix} 7 & 5 \\ 12 & 10 \\ 7 & 3 \end{bmatrix} - \begin{bmatrix} 6 & 3 \\ -4 & 3 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7-6 & 5-3 \\ 12-(-4) & 10-3 \\ 7-2 & 3-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 16 & 7 \\ 5 & 0 \end{bmatrix}$$

$$\text{and } (B-C) = \begin{bmatrix} 5 & 2 \\ 7 & 3 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 6 & 0 \\ -4 & 3 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 11 & 0 \\ -1 & -3 \end{bmatrix}$$

$$\therefore A + (B-C) = \begin{bmatrix} 2 & 3 \\ 5 & 7 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 11 & 0 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 16 & 7 \\ 5 & 0 \end{bmatrix}$$

$$\therefore (A+B) - C = A + (B-C) \text{ proved}$$

(1) (6)

Ex-(4) If $A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 2 & 4 & -1 & 3 \end{bmatrix}_{2\times 4}$ and $B = \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 2 & 3 \end{bmatrix}_{2\times 4}$ Such that

find a matrix X of order 2×4

$$A - X = 3B$$

Sol^n:- Given; $A - X = 3B$

$\therefore X = A - 3B$

$$3B = 3\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 0 & 9 \\ 3 & -3 & 6 & 9 \end{bmatrix}$$

$$\therefore A - 3B = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 2 & 4 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & 3 & 0 & 9 \\ 3 & -3 & 6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6 & 2-3 & 0-0 & 4-9 \\ 2-3 & 4-(-3) & -1-6 & 3-9 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & 0 & -5 \\ -1 & 7 & -7 & -6 \end{bmatrix}_{2\times 4}$$

Matrix Multiplication:-

Multiplication of a Matrix by a matrix:- When the number of columns of matrix A is the same as the number of rows of another matrix B, then A is said to be comformable to B for the product AB and thereby the product AB is defined and denoted by $A.B$ or AB. If $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{n\times p}$ then the order of the product AB is $m\times p$.

Note:– When the number of columns in Matrix A = The number of rows in matrix B, we say That A and B are conformable for the product AB. For example: If $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}_{2\times2}$

Then $A \times B$ or $A \cdot B = ?$ $B = \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \end{bmatrix}$

Sol^n:– A and B are conformable for the product AB, as the no. of columns in matrix A is equal to the no. of rows in matrix B and the order of the product AB will be 2×2.

$$AB = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \end{bmatrix}$$

$$= \begin{bmatrix} \overset{\text{1st row in A} \times \text{1st col}^n \text{ in B}}{(a_1 \quad b_1) \times \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}} & \overset{\text{1st row in A} \times \text{2nd col}^n \text{ in B}}{(a_1 \quad b_1) \times \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}} \\ \overset{\text{2nd row in A} \times \text{2nd col in B}}{(a_2 \quad b_2) \times \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}} & \overset{\text{2nd row in A} \times \text{2nd col}^n \text{ in B}}{(a_2 \quad b_2) \times \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}} \end{bmatrix}$$

$$AB = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}_{2\times2} \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \end{bmatrix}_{2\times2} = \begin{bmatrix} a_1c_1 + b_1c_2 & a_1d_1 + b_1d_2 \\ a_2c_1 + b_2c_2 & a_2d_1 + b_2d_2 \end{bmatrix}_{2\times2}$$

Properties of Matrix Multiplication:–

(i) The product of matrices is not commutative; $AB \neq BA$

(ii) Multiplication of matrices is associative; $(AB)\times C = A(BC)$.

(iii) If A is an $m\times n$ matrix and O is a null matrix of order $n\times m$, then $A\cdot O = O\cdot A = O$.

(iv) If A be a square matrix of order $m \times n$ and I be the unit matrix of the same order, then $AI = IA = A$.

Ex:-(1): If $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}_{2\times 2}$ and $B = \begin{bmatrix} -1 & 7 \\ 8 & 4 \end{bmatrix}_{2\times 2}$

Then find AB.

Solⁿ:- $AB = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}_{2\times 2} \times \begin{bmatrix} -1 & 7 \\ 8 & 4 \end{bmatrix}_{2\times 2}$

$$= \begin{bmatrix} 3\times(-1)+6\times 8 & 3\times 7+6\times 4 \\ 2\times(-1)+4\times 8 & 2\times 7+4\times 4 \end{bmatrix}$$

$$= \begin{bmatrix} -3+48 & 21+24 \\ -2\times 32 & 14+16 \end{bmatrix} = \begin{bmatrix} 45 & 45 \\ 30 & 30 \end{bmatrix}_{2\times 2}$$

Ex-(2) If $A = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}_{1\times 3}$ and $B = \begin{bmatrix} 5 \\ 6 \\ 2 \end{bmatrix}_{3\times 1}$ find AB.

Solⁿ:- Matrix A and B are conformable for the product AB, since the no. of columns in A is equal to the no. of rows in B and the order of product AB will be 1×1.

$$AB = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}_{1\times 3} \times \begin{bmatrix} 5 \\ 6 \\ 2 \end{bmatrix}_{3\times 1} = \begin{bmatrix} 2\times 5+3\times 6+4\times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10+18+8 \end{bmatrix} = \begin{bmatrix} 36 \end{bmatrix}_{1\times 1}$$