Q.No. 11. দুটা অশূন্য ভেক্টৰ $\vec{a}$ আৰু $\vec{b}$-ৰ বাবে দেখুওৱা যে $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$, $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$-ৰ ওপৰত লম্ব।

Sol^n. We have,

$$\left(|\vec{a}|\vec{b}+|\vec{b}|\vec{a}\right)\cdot\left(|\vec{a}|\vec{b}-|\vec{b}|\vec{a}\right)$$

$$= |\vec{a}|^2\,\vec{b}\cdot\vec{b} - \cancel{|\vec{a}||\vec{b}|\,\vec{b}\cdot\vec{a}} + \cancel{|\vec{b}||\vec{a}|\,\vec{a}\cdot\vec{b}} - |\vec{b}|^2\,\vec{a}\cdot\vec{a}$$

$$= |\vec{a}|^2|\vec{b}|^2 - |\vec{b}|^2|\vec{a}|^2$$

$\left(\because \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a} \text{ and } \vec{a}\cdot\vec{a}=|\vec{a}|^2 \text{ etc.}\right)$

$$= 0$$

$\therefore$ প্ৰদত্ত ভেক্টৰ দুটা পৰস্পৰ লম্ব (given vectors are $\perp^r$ to each other).

Q.No. 13. $\hat{a}, \hat{b}, \hat{c}$ তিনিটা একক ভেক্টৰ আৰু $\hat{a}+\hat{b}+\hat{c}=\vec{0}$, $\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}$ উলিওৱা।

Sol^n. Given, $\hat{a}+\hat{b}+\hat{c}=\vec{0} \longrightarrow (1)$

(1)-ক $\hat{a}$ ৰে ডট পূৰণ কৰি পাওঁ (Taking dot product of (1) by $\hat{a}$)

$$\hat{a}\cdot\hat{a}+\hat{a}\cdot\hat{b}+\hat{a}\cdot\hat{c}=\hat{a}\cdot\vec{0}$$

$\Rightarrow 1 + \hat{a}\cdot\hat{b} + \hat{c}\cdot\hat{a} = \vec{0} \quad [\because \hat{a}\cdot\hat{a} = 1, \ \hat{c}\cdot\hat{a} = \hat{a}\cdot\hat{c}]$

$\Rightarrow \hat{a}\cdot\hat{b} + \hat{c}\cdot\hat{a} = -1 \longrightarrow (2)$

আকৌ, (1) ক $\hat{b}$-ৰে ডট্ পূৰণ লৈ পাওঁ,
(Taking dot product of (1) by $\hat{b}$)

$\hat{b}\cdot\hat{a} + \hat{b}\cdot\hat{b} + \hat{b}\cdot\hat{c} = \hat{b}\cdot\vec{0}$

$\Rightarrow \hat{a}\cdot\hat{b} + 1 + \hat{b}\cdot\hat{c} = 0, \quad [\because \hat{b}\cdot\hat{b} = 1, \ \hat{a}\cdot\hat{b} = \hat{b}\cdot\hat{a}]$

$\Rightarrow \hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} = -1 \longrightarrow (3)$

সেইদৰে (similarly), (1) ক $\hat{c}$-ৰে ডট্ পূৰণ লৈ পাওঁ,

$\hat{c}\cdot\hat{a} + \hat{c}\cdot\hat{b} + \hat{c}\cdot\hat{c} = \hat{c}\cdot\vec{0}$

$\Rightarrow \hat{c}\cdot\hat{a} + \hat{b}\cdot\hat{c} + 1 = 0, \quad [\because \hat{a}\cdot\hat{c} = \hat{c}\cdot\hat{a}, \ \hat{c}\cdot\hat{c} = 1]$

$\Rightarrow \hat{c}\cdot\hat{a} + \hat{b}\cdot\hat{c} = -1 \longrightarrow (4)$

Now,

$(2) + (3) + (4) \Rightarrow$

$(\hat{a}\cdot\hat{b} + \hat{c}\cdot\hat{a}) + (\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c}) + (\hat{c}\cdot\hat{a} + \hat{b}\cdot\hat{c}) = -1-1-1$

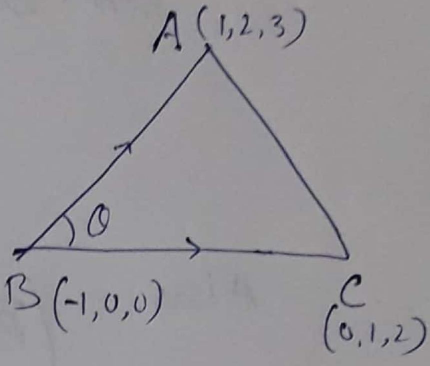
$\Rightarrow 2\,\hat{a}\cdot\hat{b} + 2\,\hat{b}\cdot\hat{c} + 2\,\hat{c}\cdot\hat{a} = -3$

$\Rightarrow 2\,[\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a}] = -3$

$\Rightarrow \hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a} = -3/2 \longleftarrow$ Ans.

Exercise 10.3

Q. No. (15). ABC ত্রিভুজৰ শীর্ষবিন্দু A, B, C ক্রমে (1, 2, 3), (−1, 0, 0) আৰু (0, 1, 2). ∠ABC উলিওৱা ।



Soln. আমি পাওঁ, (we get),

A (1,2,3) বিন্দুৰ অৱস্থান ভেক্টৰ $= \hat{i} + 2\hat{j} + 3\hat{k}$
(position vector of A)

B(−1,0,0) " " " $= -\hat{i} + 0\hat{j} + 0\cdot\hat{k} = -\hat{i}$

C (0,1,2) " " " $= 0\hat{i} + 1\cdot\hat{j} + 2\hat{k} = \hat{j} + 2\hat{k}$

p.V. = position vector
অৱস্থান ভে:

$$\therefore \overrightarrow{BA} = (\text{p.v. of } A) - (\text{p.v. of } B)$$
$$= (\hat{i} + 2\hat{j} + 3\hat{k}) - (-\hat{i})$$
$$= 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\& \ \overrightarrow{BC} = (\text{p.v. of } C) - (\text{p.v. of } B)$$
$$= (\hat{j} + 2\hat{k}) - (-\hat{i})$$
$$= \hat{i} + \hat{j} + 2\hat{k}$$

ধৰা (let), $\overrightarrow{BA}$ আৰু $\overrightarrow{BC}$-ৰ মাজৰ কোণ $\theta = \angle ABC$.

$$\therefore \overrightarrow{BA} \cdot \overrightarrow{BC} = |\overrightarrow{BA}| \cdot |\overrightarrow{BC}| \cos\theta$$

$$\Rightarrow \cos\theta = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{|\overrightarrow{AB}||\overrightarrow{BC}|} \longrightarrow (1)$$

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এখন (now),

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = \left(2\hat{i}+2\hat{j}+3\hat{k}\right)\cdot\left(\hat{i}+\hat{j}+2\hat{k}\right)$$

$$= 2+2+3.2$$

$$= 10$$

Also, $\left|\overrightarrow{BA}\right| = \sqrt{2^2+2^2+3^2} = \sqrt{17}$

& $\left|\overrightarrow{BC}\right| = \sqrt{1^2+1^2+2^2} = \sqrt{6}$

Now,

$$(1) \Rightarrow \cos\theta = \frac{10}{\sqrt{17}\sqrt{6}} = \frac{10}{\sqrt{102}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right) \leftarrow \text{Ans.}$$

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